

Unit - 1

Principle of Electric circuit.

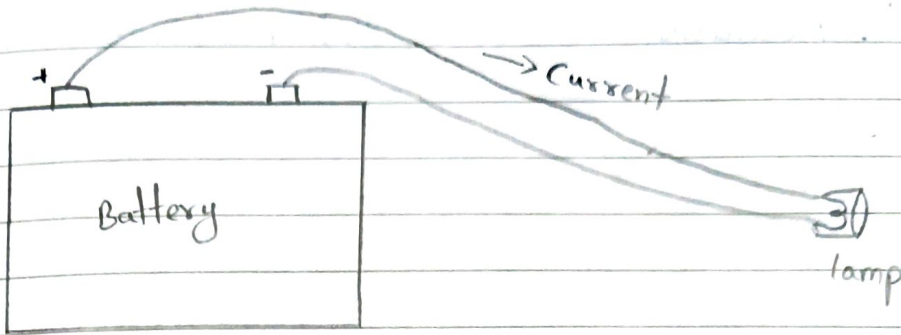


fig: Simple Electric Circuit

An Electric circuit is an interconnection of electrical elements

★ Charge:- The most basic quantity in an electrical circuit is the electric charge.

Charge is an electrical property of the atomic particles of which matters consist, It is measured in coulombs [C]

★ Current:- Electric current is the time rate of change of charge measured in ampere [A]

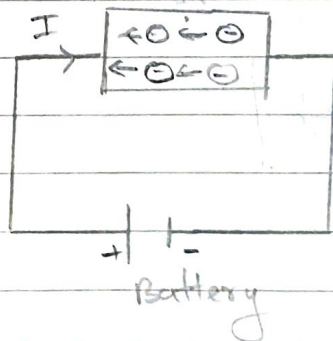
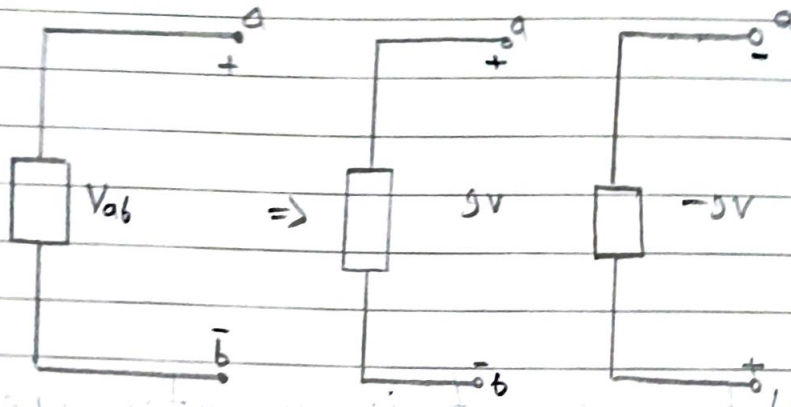


fig: Electric current due to flow of electric charge in conductors

* Voltage - Voltage (or potential difference) is the energy required to move a unit charge from a reference point (-) to another point (+). Measured in volts (V). Mathematically

$$V = \frac{dW}{dq}$$



$$[V_{ab} = -V_{ba}]$$

$$V_{ab} = 9V$$

$$V_{ab} = -9$$

Power :- Power is the time rate of expending or absorbing energy, Measured in watt [W], mathematically.

$$P = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} = Vi$$

$$[P = Vi]$$

Mathematically the relation-ship between current (i), charge [q] & time [t] is

$$[i = \frac{dq}{dt}]$$

Types of Current:-

1) Direct Current (DC)

A direct current flows only in one direction & can be constant.

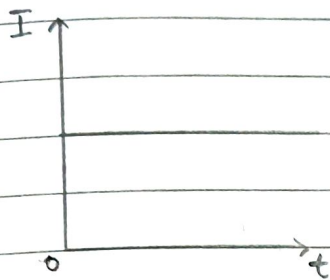


fig:- Direct current

2) Alternating Current (AC):-

An Alternating Current is a current that changes direction with respect to time.

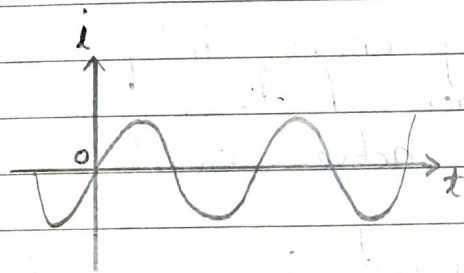
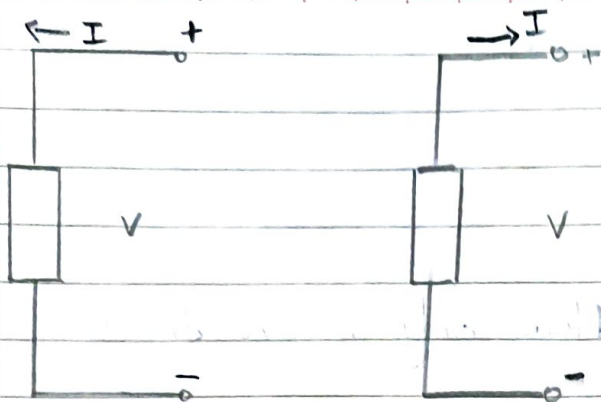


fig:- Alternating Current

★ Energy:- Energy is the capacity to do work measured in Joule (J).

The Electric power utility companies measure energy in watt hours

$$1 \text{ Wh} = 3600 \text{ J}$$



$$P = VI$$

$$P = -VI$$

a) Absorbing Power

b) Supplying Power

fig:- Powers

★ Circuit Elements :- An Element can be completely characterized by means V, I [V, i] relationship.

i) Active Elements :- When an element is capable of delivering energy independently for an infinite duration of time, then the element is called as active Element.

Ex:- Voltage Source, Current Source, transistors etc.

ii) Passive Elements :- When an element is not capable of delivering energy independently for an infinite duration of time, then the element is called as passive element.

Ex:- Resistor, capacitor, inductor etc.

★ Bi-directional element:- When an element properties and characteristics are independent of the direction of current flow, then the element is called as Bi-directional element.

Ex:- resistor, inductor, capacitor etc.

★ Uni-directional element:- When an element properties and characteristics are dependent on the direction of the current flow then the element is called as uni-directional element.

Ex:- Diode etc.



fig :- diode

v) linear & non-linear element:- Linearity is the property of an element describing a linear relationship between the excitation & response, otherwise non linear.

★ Source:-

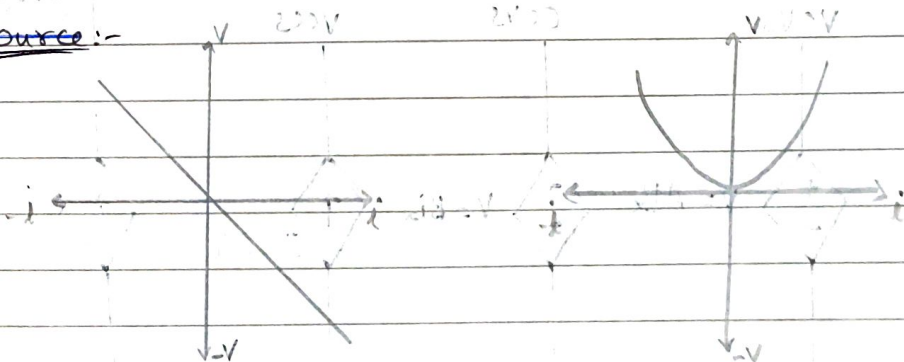


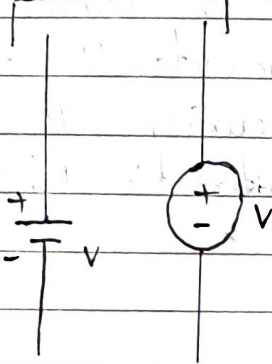
fig:- linear

(b) non-linear:-

Source :-

If the terminal voltage is completely independent of the current or the current is completely independent of the voltage, then the element is called as independent source.

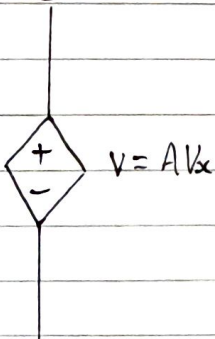
Independent voltage Source



Independent current source

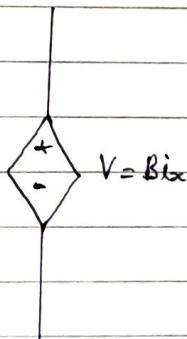
* The element or which either voltage or the current depends upon the current or voltage somewhere else in the circuit is called as dependent source.

VCVS



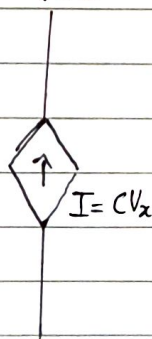
voltage control
voltage source

CCVS



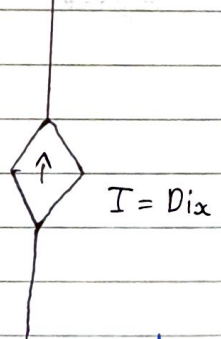
current control
voltage source

VCCS



voltage control
current source

CCCS



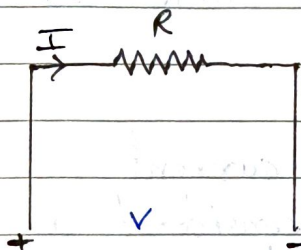
current control
current source

1.2) Passive elements / components (RLNC) :- study state behaviour in DC circuit.

1) Resistor:- If the voltage across an element is linearly proportional to the current through it then that element is called as resistor.

Resistance can be described as that property of circuit element which offers opposition to the flow of current and in doing so it converts electrical energy to heat energy.

Unit :- Ohms (Ω) or $\frac{\text{Volt}}{\text{Amp.}}$



2) we know that

$$\text{Energy (W)} = \int_0^t \text{Power (P)} dt$$

$$W = \int_0^t P dt$$

25) Hence $P = IV$ & $V = IR$

$$P = I(IR)$$

$$P = I^2 R$$

$$\therefore W = \int_0^t I^2 R dt$$

$$W = I^2 R \int_0^t dt$$

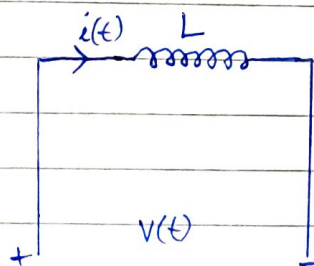
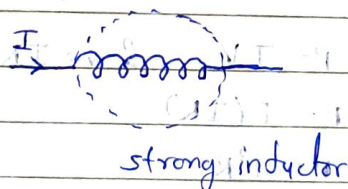
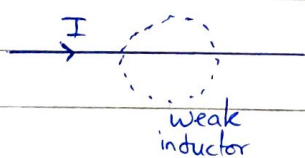
$$[W = I^2 R t] \quad \text{Imp}$$

Immediately after the circuit is connected the current & the voltage drop across the resistor will be very small both will increase until the circuit is at steady state.

★ Inductor:- If terminal voltage is proportional to the derivative of the current with respect to time then that element is called as inductor.

- Inductor is designed to store the energy in its magnetic field.
- Any conductor carrying current as inductive property and may be considered as an inductor but its inductive effect is ~~not~~ very weak & to increase its inductive effect ~~a~~ no. of turns are provided.

Unit = Henry (H)



$$V(t) = L \frac{di}{dt}$$

$$\text{Energy} = W(t) = \int P(t) dt$$

$$W(t) = \int V(t) \cdot i(t) dt$$

$$W(t) = \int L \frac{di}{dt} \cdot i dt$$

$$W(t) = L \frac{di}{dt} \int i dt$$

$$W(t) = L \frac{i^2}{2}$$

$$\left[W(t) = \frac{Li^2}{2} \right]$$

• Inductor store energy in the form of ~~energy~~ magnetic field (kinetic energy)

• Under steady state condition for the DC source inductor act as a short circuit

$$V = L \frac{di}{dt}$$

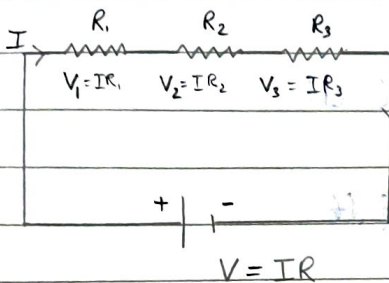
$i \rightarrow$ constant in DC

$$V = \frac{di}{dt} = 0$$

$V = 0$ (short circuit)

★ Kirchoff's voltage law (KVL)

In any closed ~~an~~ circuit or mesh the algebraic sum of all the EMF ~~an~~ or source voltage & the voltage drops is equal to zero.

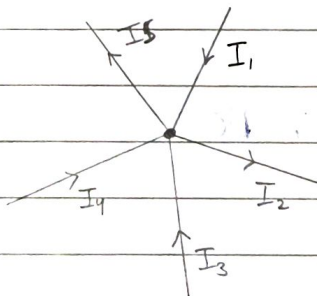


$$V - V_1 - V_2 - V_3 = 0$$

OR $V = V_1 + V_2 + V_3$

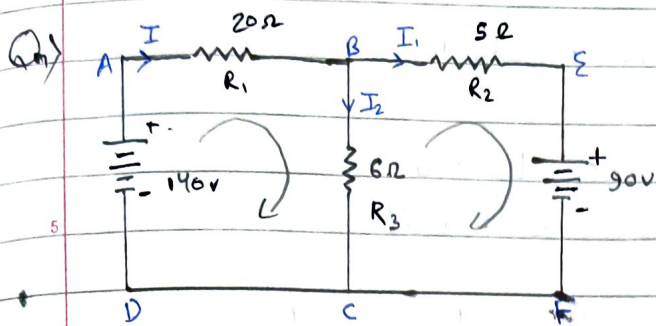
★ Kirchoff's current law (KCL)

The algebraic sum of all the incoming & outgoing currents meeting at a point or junction is 0 (zero).



$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

$$I_1 + I_2 + I_3 = I_4 + I_5$$



In the circuit shown calculate current & power in 6Ω resistance?

→ Applying kirchoff's ~~current~~ ^{voltage} law to mesh ABCDA

i.e,

$$-20(I_1 + I_2) - 6(I_2) + 140 = 0$$

$$-20I_1 - 20I_2 - 6I_2 + 140 = 0$$

$$-20I_1 - 26I_2 = -140$$

$$[20I_1 + 26I_2 = 140] \quad \text{--- (1)}$$

Applying KVL to mesh BEFCB

$$-5(I_1) - 90 + 6I_2 = 0$$

$$-5I_1 + 6I_2 = 90$$

multiplying eqn (2) by 4

$$-20I_1 + 24I_2 = 360 \quad \text{--- (3)}$$

On adding eqn (1) & (3) we get

$$20I_1 + 26I_2 = 140$$

$$-20I_1 + 24I_2 = 360$$

$$50I_2 = 500$$

$$I_2 = \frac{500}{50} \quad \text{i.e. } I_2 = 10$$

Current in 6Ω resistance = 10

2) Power in 6Ω resistance $= I_2^2 R_3$

$$= 10^2 \times 6$$

$$= 600 \text{ watt}$$

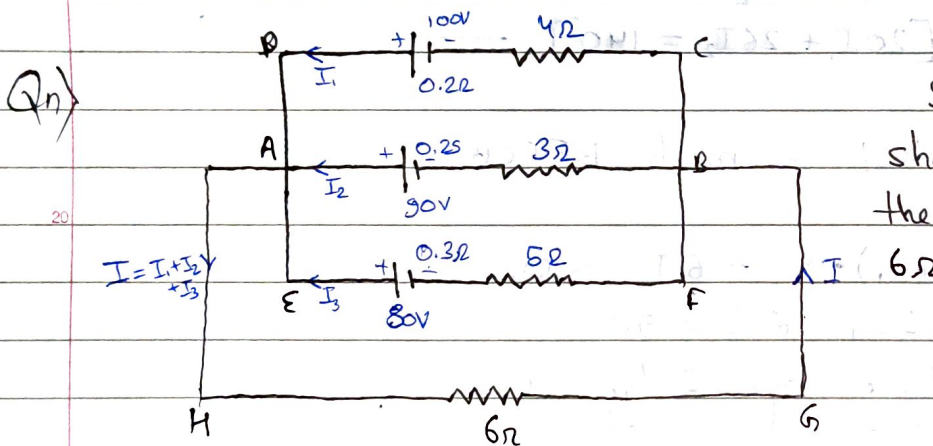
Qn) Notes:- When we go through -ve to +ve in voltage source or source (any) it can be known as -ve to +ve = Rise

Constant value should be written in +ve.



When we go through +ve to -ve in source = fall:

↓ $\frac{1}{T}$ Constant value should be written in -ve



Solve the network shown in fig. for the current through 6Ω resistor...

⇒ Applying KVL to closed circuit CDAGBC

$$-4I_1 + 100 - 0.2I_1 - 6(I_1 + I_2 + I_3) = 0$$

$$-4I_1 - 0.2I_1 - 6I_1 - 6I_2 - 6I_3 = -100$$

$$-10.2I_1 - 6I_2 - 6I_3 = -100$$

$$10.2I_1 + 6I_2 + 6I_3 = 100 \quad \text{--- (1)}$$

Applying KVL to closed circuit BAHGB

$$-3I_2 + 90 - 0.25I_2 - 6(I_1 + I_2 + I_3) = 0$$

$$-3I_2 - 0.25I_2 - 6I_1 - 6I_2 - 6I_3 = -90$$

$$-9.25I_2 - 6I_1 - 6I_3 = -90$$

$$6I_1 + 9.25I_2 + 6I_3 = 90 \quad \text{--- (2)}$$

Applying KVL to closed circuit FEAHGBF

$$-5I_3 + 80 - 0.32I_3 - 6(I_1 + I_2 + I_3) = 0$$

$$-5I_3 - 0.32I_3 - 6I_1 - 6I_2 - 6I_3 = -80$$

$$-11.32I_3 - 6I_1 - 6I_2 = -80$$

$$6I_1 + 6I_2 + 11.32I_3 = 80 \quad \text{--- (3)}$$

from solving the all 3 eqn we get

$$I_1 = 6.21 \text{ A}, \quad I_2 = 4.95 \text{ A}, \quad I_3 = 1.15 \text{ A}$$

$\therefore$ According to question.

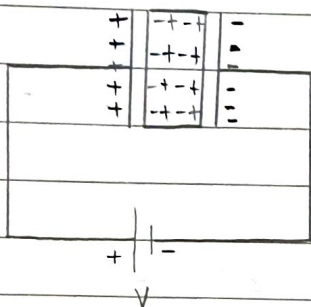
$$\text{Current through } 6\Omega \text{ resistance} = I_{6\Omega} = I_1 + I_2 + I_3$$

$$\begin{aligned} I_{6\Omega} &= 6.21 + 4.95 + 1.15 \\ &= 12.31 \text{ Amp} \end{aligned}$$

Hence the current through 6Ω resistance = 12.31 Amp

★ Source transformation Capacitor:-

- If the terminal voltage is proportional to the integral of current with respect to time then the element is called as capacitor.
- Capacitor is designed to store the energy in its electric field.



$$Q \propto V$$

$$Q = CV$$

$$i = \frac{dQ}{dt}$$

$$i = C \frac{dV}{dt}$$

$$i = C \frac{dv}{dt}$$

$$\left[V = \frac{1}{C} \int i dt \right]$$

Energy:- $w(t) = \int v(t) \cdot i(t) dt$

$$= \int V \cdot C \frac{dV}{dt} dt$$

$$= C \frac{V^2}{2}$$

$$\left[W = \frac{1}{2} CV^2 \right]$$

- Capacitor stores energy in the form of electric field.
- For DC supply under steady state condition, capacitor act as a open circuit.

$$i = C \frac{dV}{dt}$$

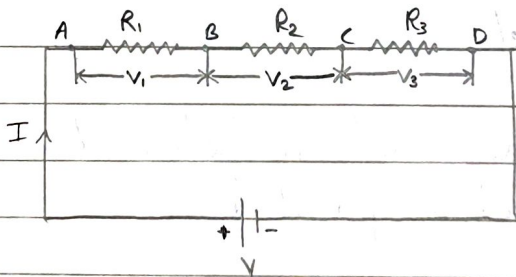
in dc $V = \text{constant value}$

$$\therefore i = C \times 0$$

$[i = 0]$ works as open circuit.

★ Simple series & parallel resistive circuit

- Resistance in series:- When some conductors having resistance R_1, R_2 & R_3 etc are joined in end-on-end as shown in fig. below, they are said to be connected in series.



It can be proved that the equivalent resistance or total resistance between points A & D is equal to the sum of the three individual resistances.

$$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

But $V = IR$

where, $R \Rightarrow$ Equivalent Resistance (R_{eq})

$$IR_{eq} = IR_1 + IR_2 + IR_3$$

$$IR_{eq} = I(R_1 + R_2 + R_3)$$

$$[R_{eq} = R_1 + R_2 + R_3]$$

Voltage divider rule:-

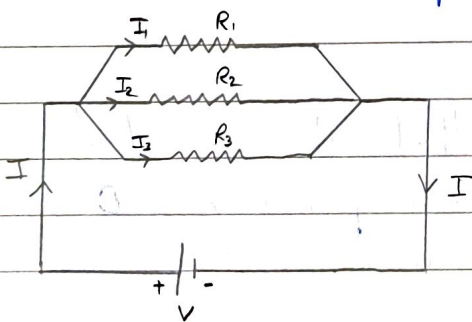
$$V_1 = V \cdot \frac{R_1}{R_{eq}}$$

$$V_2 = V \cdot \frac{R_2}{R_{eq}}$$

$$V_3 = V \cdot \frac{R_3}{R_{eq}}$$

★ Resistance in parallel

The resistances as joint in below figure are said to be connected in parallel.



$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

But $I = \frac{V}{R}$
 where, $R \rightarrow R_{eq}$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\left[\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]$$

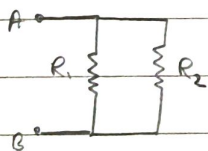
Current division rule:-

$$I_1 = I \times \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_2 = I \times \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_3 = I \times \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Quick formula for solving two resistance in parallel



$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_2 + R_1}{\frac{R_1 R_2}{R_1 + R_2}}$$

$$\therefore \left[R_{AB} = \frac{R_1 R_2}{R_1 + R_2} \right]$$

Q1) Find equivalent resistance b/w A & B.



⇒ Parallel equiv resistance
between ~~R~~ C to D

$$R_{CD} = \frac{2 \times 2}{2 + 2}$$

$$= \frac{4}{4} = 1\Omega$$

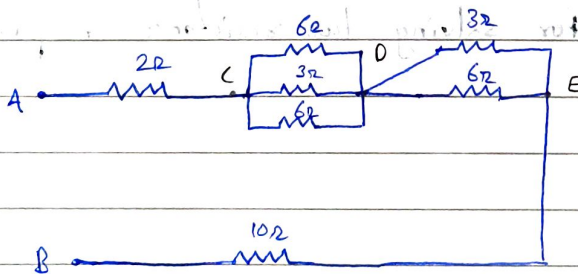


~~R~~ Equivalent resistance = $R_1 + R_2$

$$= 2 + 1$$

$$= 3\Omega \quad \text{Answer}$$

② Find equivalent resistance between A & B.



⇒ Step ① parallel resistance between DE

$$R_{DE} = \frac{3 \times 6}{3 + 6}$$

$$= \frac{18}{9} = 2\Omega$$

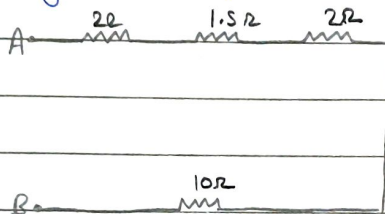
Step ② Solving parallel resistance between C & D

$$R_{CD} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{CD} = \frac{1}{6} + \frac{1}{3} + \frac{1}{6} = \frac{1+2+1}{6} = \frac{4}{6}$$

$$R_{CD} = \frac{6}{4} = \frac{3}{2} \Rightarrow 1.5 \Omega$$

Step ③ solving series eq resistance between A & B



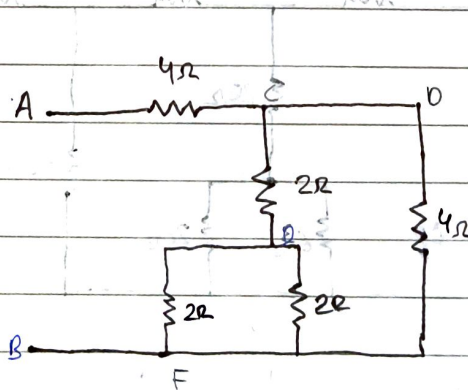
$$R_{AB} = R_1 + R_2 + R_3 + R_4$$

$$= 2 + 1.5 + 2 + 10$$

$$[R_{AB} = 15.5 \Omega]$$

Answer

Qn3) Find R_{AB}



⇒ Step ① parallel resistance between E & F

$$R_{EF} = \frac{2 \times 2}{2 + 2}$$

$$= \frac{4}{4} = 1$$

Step ② solving series resistance between C & F

$$R_{CF} = 2 + 1 \\ = 3 \Omega$$

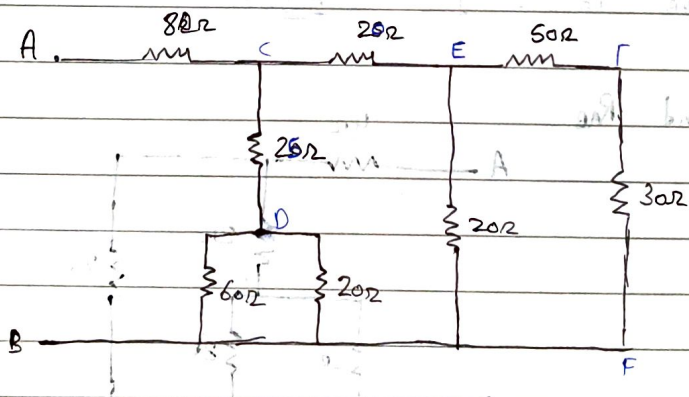
Step ③ solving equivalent resistance from A to B

$$R_{AB} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{AB} = 4 + \frac{3 \times 4}{3 + 4} \\ = 4 + \frac{12}{7}$$

$$[R_{AB} = 5.714 \Omega]$$

Q10 Find R_{AB}



⇒ Step ① Parallel resistance btwn D to B

$$R_{DB} = \frac{R_1 * R_2}{R_1 + R_2} = \frac{60 * 20}{60 + 20}$$

$$\Rightarrow \frac{1200}{80} = 15 \Omega$$

Step ② Calculating R_{CB}

$$R_{CB} = R_1 + R_2$$

$$= 25 + 15$$

$$R_{CB} = 40 \Omega$$

Step ③ Calculating R_{EF}

$$R_{EF} = 50 + 30$$

$$= 80 \Omega$$

Step ④ Calculating R_{EB}

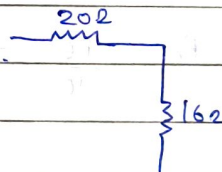
$$R_{EB} = \frac{80 \times 20}{80 + 20}$$

$$R_{EB} = 16 \Omega$$

Step ⑤ Calculating circuit in series

$$R_{eq} = 20 + 16$$

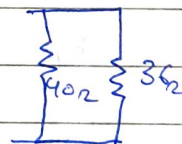
$$= 36 \Omega$$



Step ⑥ Calculating circuit in parallel

$$R_{||} = \frac{40 \times 36}{40 + 36}$$

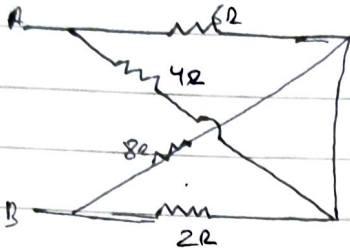
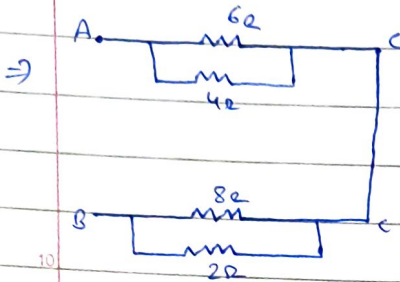
$$= 18.94 \Omega$$



Step ⑦ Calculating R_{AB}

$$R_{AB} = 80 + 18.94 \Rightarrow 98.94 \Omega$$

Question

Calculate R_{AB} ?

$$R_{AC} = \frac{6 \times 4}{6 + 4} = \frac{6 \times 4}{6 + 4}$$

$$R_{AC} = \frac{24}{10} = 2.4 \Omega$$

$$R_{BC} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{8 \times 2}{8 + 2}$$

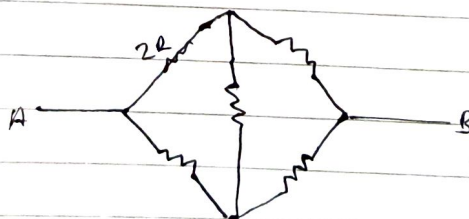
$$R_{BC} = \frac{16}{10} \Rightarrow 1.6 \Omega$$

Calculating R_{AB}

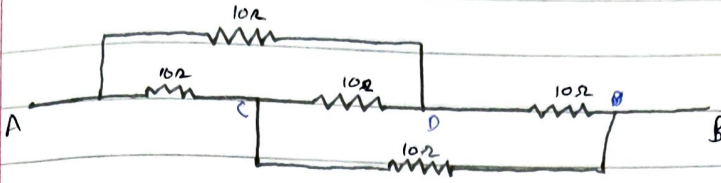
$$R_{AB} = R_1 + R_2$$

$$= 2.4 + 1.6$$

$$[R_{AB} = 4 \Omega]$$

6h) Find R_{AB} 

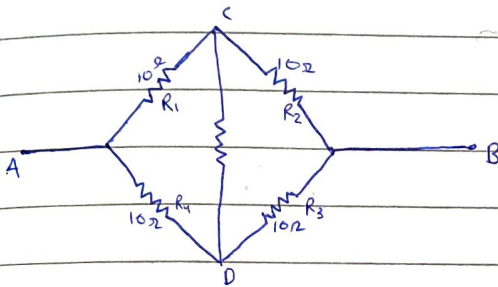
(Que)

calculate R_{AB}

⇒ Hence the bridge is under balanced condition.

∴ $R_{CP} = 0$ ~~under~~ under balanced condition.

10 After simplifying the circuit:-



Calculating series resistance

$$R_{top} \Rightarrow R_1 + R_3$$

$$\Rightarrow 10 + 10$$

$$\Rightarrow 20\Omega$$

$$R_{bottom} \Rightarrow R_2 + R_4$$

$$\Rightarrow 10 + 10$$

$$\Rightarrow 20\Omega$$

Calculating equivalent circuit R_{AB}

$$\frac{1}{R_{AB}} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}}$$

$$\Rightarrow \frac{1}{20} + \frac{1}{20}$$

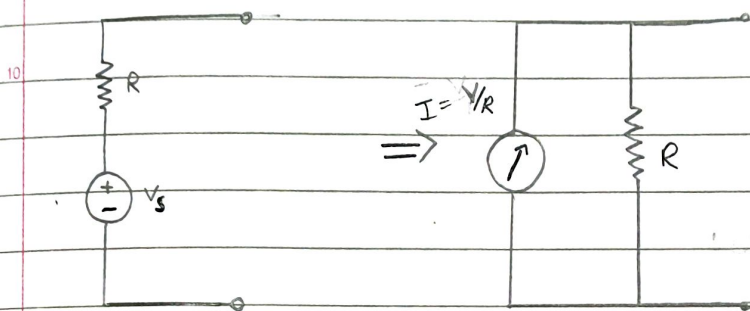
$$\frac{1}{R_{AB}} \Rightarrow \frac{2}{20}$$

$$\frac{1}{R_{AB}} = \frac{1}{10}$$

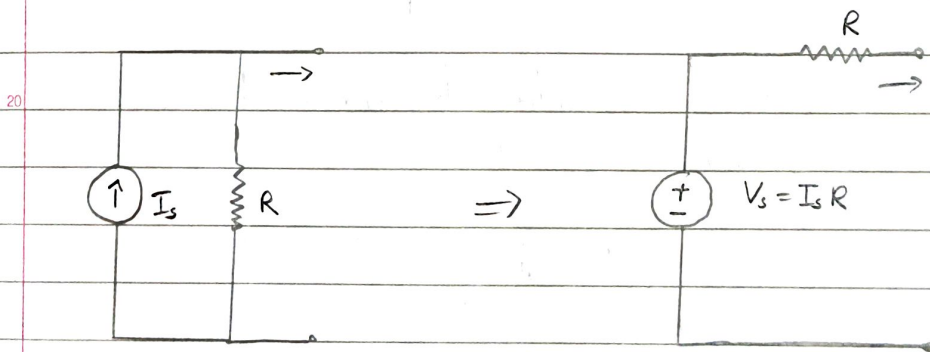
$$R_{AB} = 10 \Omega$$

★ Source transformation:-

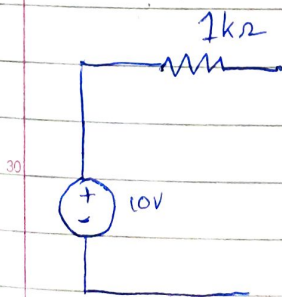
A practical voltage source can be transformed to an equivalent practical current source. By replacing the source voltage to its equivalent current source is known as ~~current~~ source transformation.



Similarly a practical current source can be transformed into a voltage source as given below:-



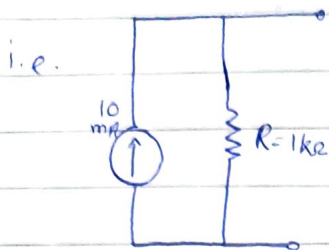
Q2) Convert voltage source to current source:-



$$I_s = \frac{V}{R}$$

$$I_s = \frac{10}{1k\Omega}$$

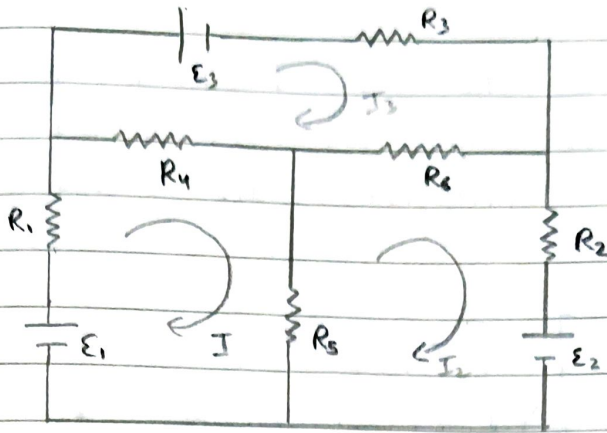
$$= 10\text{mA or } 0.01\text{A}$$



* Mesh Analysis:-

- In this method the mesh currents are considered and each mesh is assigned ~~are~~ a different mesh current. Mesh current is assumed to flow in the clock wise without splitting at the junction.
- Kirchoff's law is applied to write the equations in terms of unknown current.
- Branch current ~~is~~ can be determined with the help of following steps.

step 1) The whole network is divided into no. of mesh. Each mesh is assigned a mesh current having a continuous path as given below:



Step 2) Each mesh current is drawn in the clockwise direction.

Step 3) Write KVL equation for each mesh.

Step 4) Number of equation must be equal to no. of unknown quantities. Solve the eqn by any method and determine the mesh current.

Step 5) From the mesh current determine the branch current.

Step 6) After applying KVL

$$E_1 = (R_1 + R_2 + R_3) I_1 - R_5 I_2 - R_4 I_3$$

$$-E_2 = -R_5 I_1 + (R_2 + R_5 + R_6) I_2 - R_6 I_3$$

$$-E_3 = -R_4 I_1 - R_6 I_2 + (R_3 + R_4 + R_6) I_3$$

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_5 & -R_4 \\ -R_5 & R_2 + R_5 + R_6 & -R_6 \\ -R_4 & -R_6 & R_3 + R_4 + R_6 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ -\varepsilon_2 \\ -\varepsilon_3 \end{bmatrix}$$

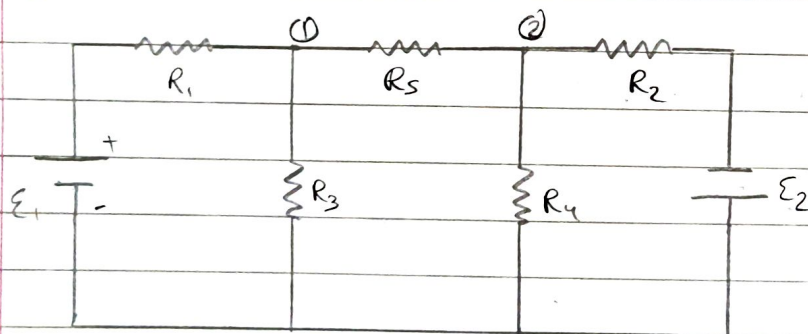
$$\Downarrow$$

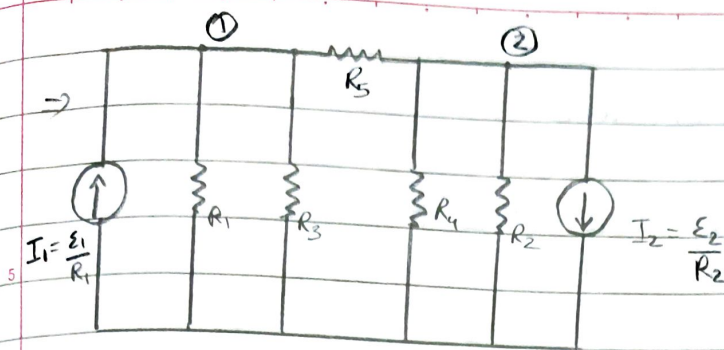
$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \varepsilon V_1 \\ \varepsilon V_2 \\ \varepsilon V_3 \end{bmatrix}$$

- Step 7) After making the matrix equation the mesh current I_1, I_2 & I_3 can be calculated with the help of cramer's rule $I_1 = \frac{\Delta_1}{\Delta}$, $I_2 = \frac{\Delta_2}{\Delta}$, $I_3 = \frac{\Delta_3}{\Delta}$

★ Node voltage method:-

In this method one of the node is taken as the reference node. The voltage of remaining $(N-1)$ nodes with respect to the reference node from an independent set of variables that implicit satisfy KVL equation $(N-1)$ KCL equations are written down at the nodes.





• For algebraic equations in $(N-1)$ nodal voltage once the nodal voltage are obtained from this in writing the nodal equation, it is convenient to convert all resistance values to conductance before proceeding the analysis

• Step 1) first of all convert all the voltage sources into current sources

• Step 2) Apply KCL at node 1 & 2 respectively

At node 1) Applyin kcl

$$-\frac{V_1}{R_1} - \frac{V_1}{R_3} + \frac{E_1}{R_1} - \frac{(V_1 - V_2)}{R_5} = 0$$

$$-\frac{V_1}{R_1} - \frac{V_1}{R_3} - \frac{V_1}{R_5} + \frac{V_2}{R_5} = -\frac{E_1}{R_1}$$

$$\frac{V_1}{R_1} + \frac{V_1}{R_3} + \frac{V_1}{R_5} - \frac{V_2}{R_5} = \frac{E_1}{R_1}$$

$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5} \right) - V_2 \left(\frac{1}{R_5} \right) = E_1 \left(\frac{1}{R_1} \right)$$

$$V_1 (G_1 + G_3 + G_5) - V_2 (G_5) = E_1 (G_1) \quad \text{————— ①}$$

At node 2) Applying KCL

$$-\frac{V_2}{R_2} - \frac{V_2}{R_4} - \frac{V_2}{R_5} - \frac{(V_2 - V_1)}{R_5} = 0$$

$$-\frac{V_2}{R_2} - \frac{V_2}{R_4} - \frac{V_2}{R_5} + \frac{V_1}{R_5} = \frac{\Sigma_2}{R_2}$$

$$\frac{V_1}{R_5} - \frac{V_2}{R_2} - \frac{V_2}{R_4} - \frac{V_2}{R_5} = \frac{\Sigma_2}{R_2}$$

$$V_1 \left(\frac{1}{R_5} \right) - V_2 \left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} \right) = \Sigma_2 \left(\frac{1}{R_2} \right)$$

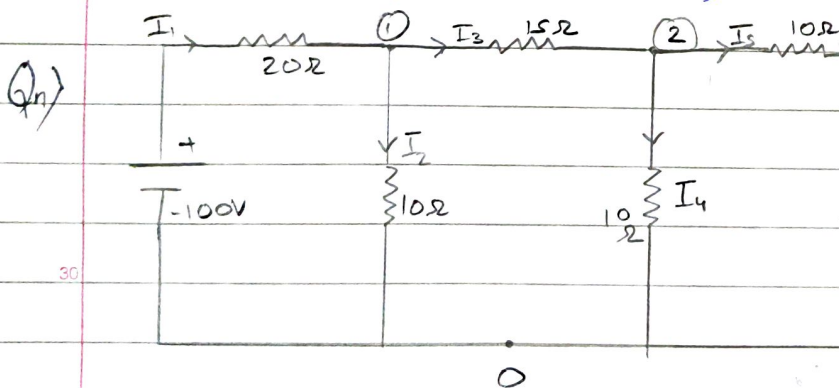
$$V_1 (G_5) - V_2 (G_2 + G_4 + G_5) = \Sigma_2 (G_2)$$

$$-V_1 (G_5) + V_2 (G_2 + G_4 + G_5) = -\Sigma_2 G_2 \quad \text{--- (2)}$$

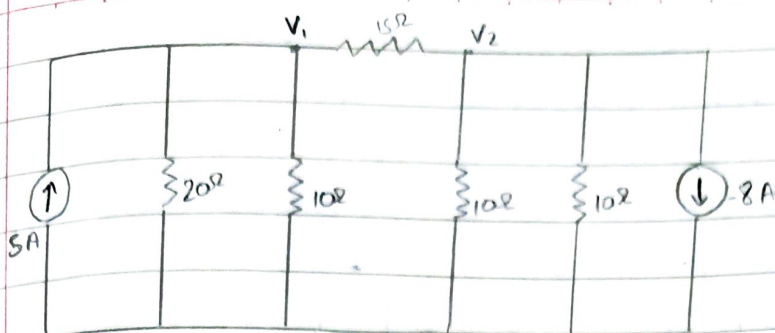
equation ① & ② can be written in matrix form as

$$\begin{bmatrix} G_1 + G_3 + G_5 & -G_5 \\ -G_5 & G_2 + G_4 + G_5 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 G_1 \\ -\Sigma_2 G_2 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Sigma I_1 \\ \Sigma I_2 \end{bmatrix}$$



Find the currents in the various branches of the circuit shown by nodal analysis.



⇒ Applying KCL at node 1

$$-\frac{V_1}{20} - \frac{V_1}{10} + 5 - \frac{(V_1 - V_2)}{15} = 0$$

$$-\frac{V_1}{20} - \frac{V_1}{10} - \frac{V_1}{15} + \frac{V_2}{15} = -5$$

$$-V_1 \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{15} \right) + V_2 \left(\frac{1}{15} \right) = -5$$

$$V_1 \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{15} \right) - V_2 \left(\frac{1}{15} \right) = 5 \quad \text{--- (1)}$$

Apply KCL at node 2

$$-\frac{V_2}{10} - \frac{V_2}{10} - 8 - \frac{(V_2 - V_1)}{15} = 0$$

$$-\frac{V_2}{10} - \frac{V_2}{10} - \frac{(V_2 + V_1)}{15} = 8$$

$$-V_2 \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{15} \right) + \frac{V_1}{15} = 8$$

$$-\frac{V_1}{15} + V_2\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{15}\right) = -8 \quad \text{--- (2)}$$

$$G_{np} = \frac{1}{20} + \begin{bmatrix} \frac{1}{20} + \frac{1}{10} + \frac{1}{15} & \frac{1}{15} \\ -\frac{1}{15} & \frac{1}{10} + \frac{1}{10} + \frac{1}{15} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$$G_{11} = \frac{1}{20} + \frac{1}{10} + \frac{1}{15} = \frac{13}{60}$$

$$G_{12} = \frac{1}{15}$$

$$G_{21} = -\frac{1}{15}$$

$$G_{22} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = +\frac{4}{15}$$

$$\frac{13}{60} V_1 - \frac{1}{15} V_2 = 5$$

$$-\frac{1}{15} V_1 + \frac{4}{15} V_2 = -8$$

$$\begin{bmatrix} \frac{13}{60} & \frac{1}{15} \\ -\frac{1}{15} & \frac{4}{15} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$$V_1 = 15V$$

$$V_2 = -26.25V$$

30 Calculating the current in branch

$$I_1 = \frac{100 - V_1}{20} = \frac{100 - 15}{20}$$

$$[I_1 = 4.25 \text{ A}]$$

$$I_2 = \frac{V_1}{10} = \frac{15}{10}$$

$$[I_2 = 1.5 \text{ A}]$$

$$I_3 = \frac{V_1 - V_2}{15} = \frac{15 - (-26.25)}{15}$$

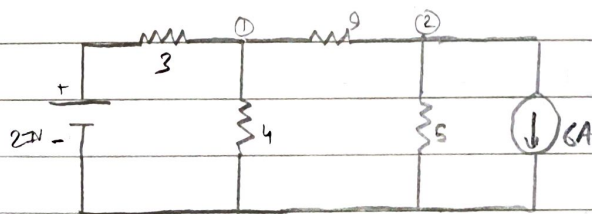
$$[I_3 = 2.75 \text{ A}]$$

$$I_4 = \frac{V_2}{10} = \frac{-26.25}{10}$$

$$[I_4 = -2.625 \text{ A}]$$

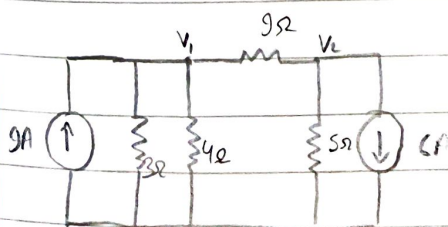
$$I_5 = \frac{V_2 + 80}{10} = \frac{-26.25 + 80}{10}$$

$$[I_5 = 5.37 \text{ A}]$$



Using nodal voltage method
compare the power dissipated in
9Ω:

solution



Applying KCL at node 1:-

$$-\frac{V_1}{3\Omega} - \frac{V_1}{4\Omega} + 9 - \frac{(V_1 - V_2)}{9} = 0$$

$$-\frac{V_1}{3\Omega} - \frac{V_1}{4\Omega} + \frac{V_1}{9} + \frac{V_2}{9} = -9$$

$$-V_1 \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{9} \right) + V_2 \left(\frac{1}{9} \right) = -9$$

$$V_1 \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{9} \right) - V_2 \left(\frac{1}{9} \right) = 9 \quad \text{--- (1)}$$

Applying KCL at node 2:-

$$-\frac{V_2}{5} - 6 - \frac{(V_2 - V_1)}{9} = 0$$

$$-\frac{V_2}{5} - \frac{V_2}{9} + \frac{V_1}{9} = 6$$

$$-\frac{V_1}{9} + \frac{V_2}{5} + \frac{V_2}{9} = -6$$

$$-V_1 \left(\frac{1}{9} \right) + V_2 \left(\frac{1}{5} + \frac{1}{9} \right) = -6 \quad \text{--- (2)}$$

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Sigma I_1 \\ \Sigma I_2 \end{bmatrix}$$

$$G_{11} = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{9} \right) = \frac{25}{36}$$

$$G_{12} = -\frac{1}{9}$$

$$G_{21} = -\frac{1}{9}$$

$$G_{22} = \left(-\frac{1}{5} + \frac{1}{9}\right) = \frac{14}{45}$$

$$\begin{bmatrix} \frac{25}{36} & \frac{1}{9} \\ -\frac{1}{9} & \frac{14}{45} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$$

$$[V_2 = -15.54]$$

$$[V_1 = 10.47]$$

Power at 9Ω resistance Therefore the current in 9Ω resistor =

$$\text{current at } 9\Omega \text{ resistance } I_{9\Omega} = \frac{V_1 - V_2}{9}$$

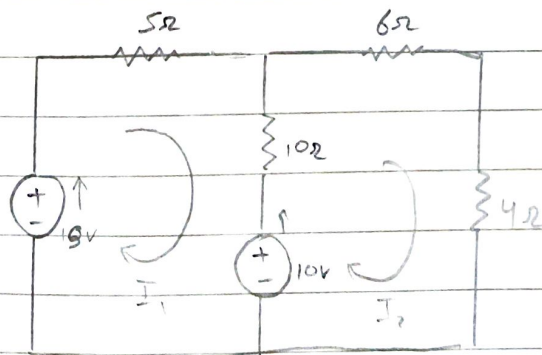
$$= \frac{10.47 + 15.54}{9}$$

$$I_{9\Omega} = 2.89 \text{ A}$$

Therefore power at 9Ω resistance $P = I^2 R$

$$= (2.89)^2 \times 9$$

$$[P = 75.16 \text{ watt}]$$



=> Applying KVL at mesh no. 1

Standard Matrix format

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \mathcal{E}V_1 \\ \mathcal{E}V_2 \end{bmatrix}$$

At mesh 1

$$15I_1 - 10I_2 = 15 - 10$$

$$15I_1 - 10I_2 = 5$$

$$3I_1 - 2I_2 = 1$$

At mesh 2

$$-10I_1 + 20I_2 = 10$$

$$-I_1 + 2I_2 = 1$$

In matrix form

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Applying crammers rule:-

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix}$$

$$\Delta = 3 \times 2 - (-2 \times -1)$$

$$\Delta = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$$

$$\Delta_1 = 1 \times 2 - (-2 \times 1)$$

$$= 4$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 \times 1 - 1 \times (-1)$$

$$= 4$$

Applying Cramer's rule

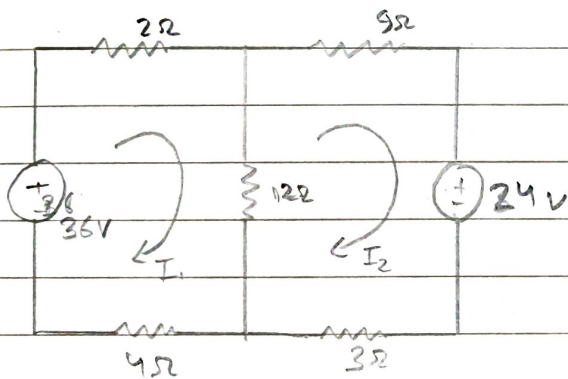
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4}{4}$$

$$[I_1 = 1 \text{ A}]$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{4}{4}$$

$$[I_2 = 1 \text{ A}]$$

2h)



⇒ Standard matrix format

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \Sigma V_1 \\ \Sigma V_2 \end{bmatrix}$$

At mesh 1:

$$18I_1 - 12I_2 = 36$$

$$3I_1 - 2I_2 = 6$$

At mesh 2

$$-12I_1 + 24I_2 = 24$$

$$-I_1 + 2I_2 = 2$$

In matrix form

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix}$$

$$\Delta = 3 \times 2 - (-2 \times -1)$$

$$\Delta = 6 - 2$$

$$\Delta = 4$$

$$\Delta_1 = \begin{vmatrix} 6 & -2 \\ 2 & 2 \end{vmatrix} = 6 \times 2 - (-2 \times 2)$$

$$= 12 + 4 = 16$$

$$\Delta_1 = 16$$

$$\Delta_2 = \begin{vmatrix} 3 & 6 \\ -1 & 2 \end{vmatrix} = 3 \times 2 - (6 \times -1)$$

$$= 6 + 6$$

$$\Delta_2 = 12$$

Applying cramer's rule

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{16}{4} = 4A$$

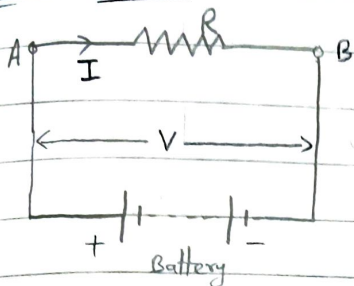
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{12}{4} = 3A$$

Current across 12Ω resistance

$$[I_1 - I_2 = 4 - 3A = 1A]$$

$$[I_{12\Omega} = 1A]$$

* Ohm's Law:-



Whenever an electric potential difference (V) is applied across a conductor or circuit as shown in figure, some current ' I ' flows through it. The flow of current is opposed by the resistance (R) of the conductor or circuit. This relation was expressed first of all by ~~T~~ George Simon ohm, a German scientist & is called ohm's law.

Ohm's law states that the current flowing between any two points of a conductor or circuit is directly proportional to the potential difference across them, provided physical condition i.e. temperature etc. do not change.

$$I \propto V$$

The ratio of potential difference across any two points of a conductor to the current flowing between them is always constant.

This constant is called resistance (R) of the conductor or circuit.

$$\frac{V}{I} = R$$

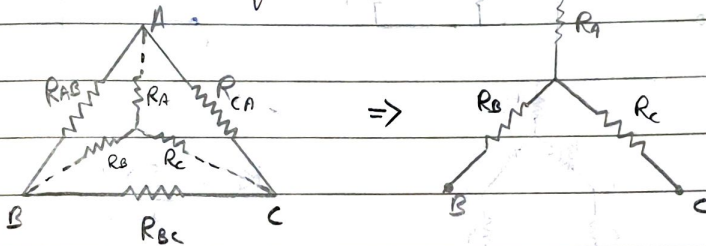
It is also written as:-

$$V = IR \text{ or } I = \frac{V}{R}$$

Unit-2

Circuit Analysis & Network theorem2.1: Star / Delta transformation of passive network:

- ★ There are some networks in which the resistances are neither in series nor in parallel. In such situation it is not possible to simplify the network by series & parallel circuit rules. However converting delta network into star and star network into delta often simplifies the network & makes it possible to apply series parallel circuit techniques.

v) Delta to star transformation:-

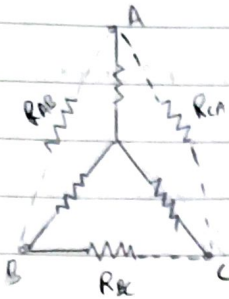
$$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{BC} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Note:- ~~Any~~ In any arm of star connection = product of two adjacent arm of delta divided sum of arm of delta.

* Star to Delta transformation:-

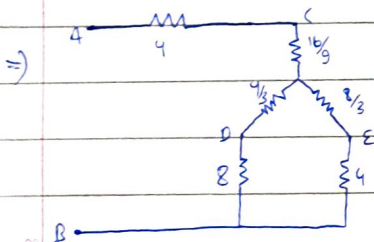
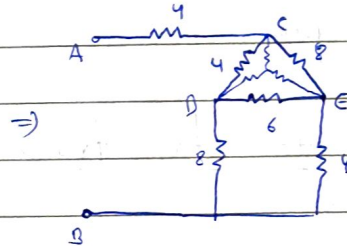
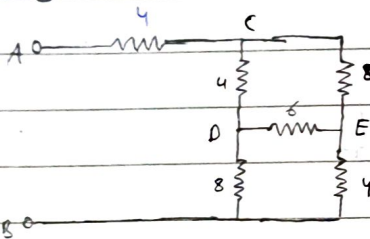


$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

* Find the input resistance of the circuit b/w the pt. A & B



$$R_C = \frac{8 \times 4}{4 + 8 + 6} = \frac{32}{18} = \frac{16}{9}$$

$$R_D = \frac{6 \times 4}{4 + 8 + 6} = \frac{24}{18} = \frac{12}{9}$$

$$R_E = \frac{6 \times 8}{4 + 8 + 6} = \frac{48}{18} = \frac{8}{3}$$

Step ① calculating resistance in series A to C

$$R_s = 4 + \frac{16}{9} = \frac{52}{9}$$

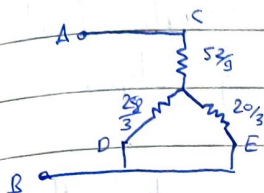
calculating resistance in series B to D

$$R_s = \frac{4}{3} + 8 = \frac{28}{3}$$

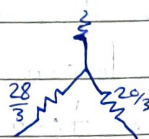
Calculating resistance in series from B to C



$$R_s = \frac{8}{3} + 4 = \frac{8+12}{3} = \frac{20}{3}$$

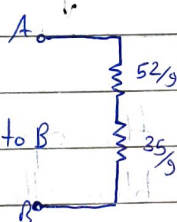


Calculating parallel resistance



$$R_{||} = \frac{\frac{28}{3} \times \frac{20}{3}}{\frac{28}{3} + \frac{20}{3}} = \frac{35}{9} \Omega$$

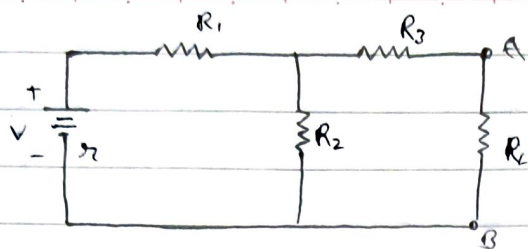
Calculating series resistance A to B



$$R_{AB} = \frac{52}{9} + \frac{35}{9}$$

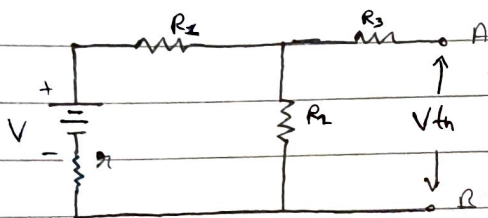
$$= \frac{29}{3} \Omega$$

* Thevenin's theorem:- The current flowing through a resistor connected across any two terminals of a network replaced by an equivalent circuit having a voltage source V_{th} in series with a resistor R_{th} where; V_{th} is open circuit voltage between required two terminals called thevenin voltage & R_{th} is the equivalent resistance of the network as seen from two terminals with all other source replaced by their internal resistance called thevenin's resistance



To determine the current through load resistance R_L , proceed with the following steps.

Step 1:- Remove the resistance R_L in which the current is to be determined, thus creating an open circuit between terminals A & B shown in figure.



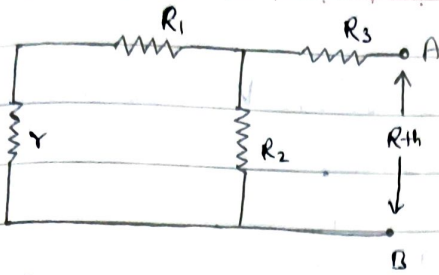
Step 2:- Determine the open circuit voltage (Thevenin voltage) V_{th} between the terminals A & B.

• Voltage across $R_2 = I_2 R_2$

$$V_{th} = \left(\frac{V}{r + R_1 + R_2} \right) \times R_2$$

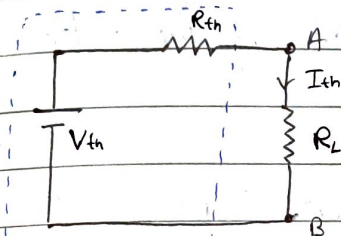
$$\boxed{V_{th} = \frac{V R_2}{r + R_1 + R_2}}$$

Step 3:- Replace the source battery by its internal resistance & determine the resistance R_{th} .



$$R_{th} = \frac{(r + R_1)R_2}{r + R_1 + R_2} + R_3$$

Step 4) Replace the entire network by a single thevenin voltage source having an EMF (V_{th}) & terminal resistance (R_{th}) as shown in figure:



↓ Thevenin's equivalent circuit

Step 5) Connect the load resistance R_L back to its terminal A & B from where it was removed.

Step 6) Determine the current flowing through the load resistance R_L by applying Ohm's law.

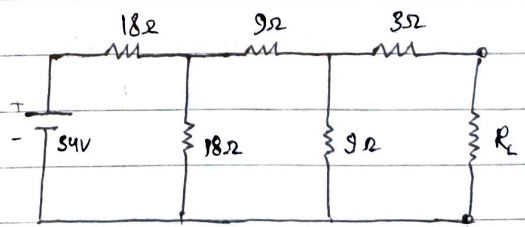
Applying Ohm's law

$$V = IR$$

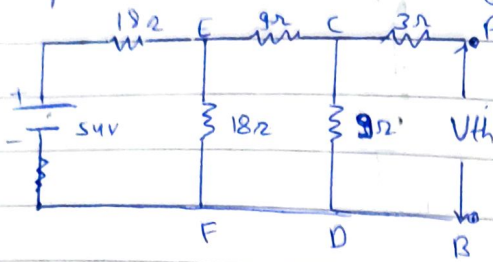
$$I = \frac{V_{th}}{R_{th} + R_L}$$

Q7) for a network shown in fig determine the current flowing through R_L when its value of R_L is

1) 3Ω 2) 6Ω 3) 9Ω



Solution $\Rightarrow$ Step 1:- To determine voltage V_{th} , load resistance is removed.



$$V_{th} = \frac{V}{R_{eq}}$$

$$R_{eq} = 18 + \frac{(9+9)18}{9+9+18} = 27 \Omega$$

The $V_{th} = 54$ current supplied by the battery $= \frac{54}{27}$

$$I = 2 \text{ Amp}$$

Current flowing through circuit, ECFD or CD =

$$= \frac{2 \times 18}{(9+9)+18} = 1 \text{ Amp}$$

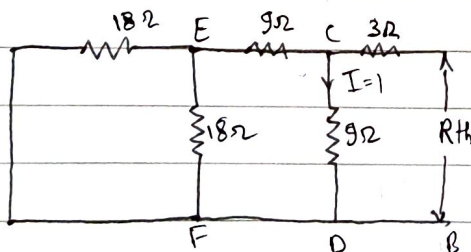
Thevenin voltage across AB or CD

By ohm's law:- $V = IR$

$$V_{th} = 1 \times 9$$

$$V_{th} = 9V$$

Step 2)

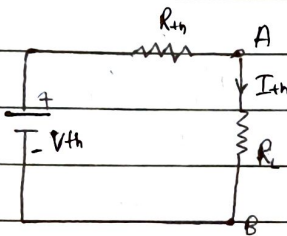


To determine thevenin's resistance the source is replaced by its internal resistance or zero in this case.

$$R_{th} = \frac{\left[\frac{18 \times 18}{18 + 18} + 9 \right] \times 9}{\left[\frac{18 \times 18}{18 + 18} + 9 \right] + 9} + 3$$

$$R_{th} = 9 \Omega$$

Step 3) To determine current flowing through load resistance R_L we redraw thevenin's equivalent circuit



$$I_{th} = \frac{V_{th}}{R_{th} + R_L}$$

When $R_L = 3$

$$I_{th} = \frac{9}{9 + 3} = 0.75 A$$

When $R_L = 6$

$$I_{th} = \frac{9}{9 + 6} = 0.6 A$$

When $R_L = 9$

$$I_{th} = \frac{9}{9 + 9} = 0.5 A$$

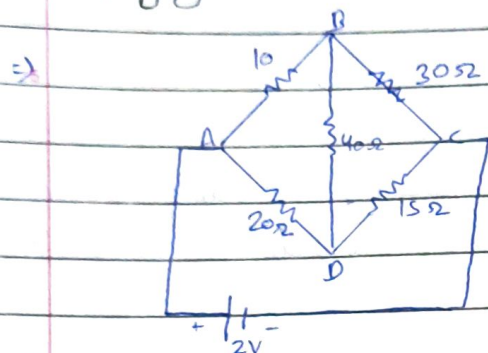
Qn) A bridge network ABCD is arranged as follows:-

Resistance b/w terminals A-B, B-C

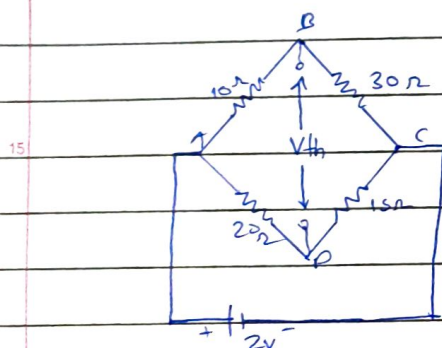
, C-D, D-A and B-D are 10, 30, 15, 20 & 40 Ω

A 2V battery of negligible internal resistance is connected between terminals A & C determine the value

and direction of the current in 40Ω resistor by applying thevenin theorem.



$\Rightarrow$ Step ① :- To determine thevenin's voltage (V_{th}), we remove 40Ω resistor then redraw the circuit



20 Current in branch ABC, $I_1 = \frac{2}{10+30} = \frac{2}{40} = 0.05A$

$$I_2 = \frac{2}{20+15} = \frac{2}{35} = 0.057A$$

Voltage drop at 10Ω resistance = $I_1 \times 10 = 0.5V$

25 Voltage drop at 20Ω resistance = $I_2 \times 20 = 1.14V$

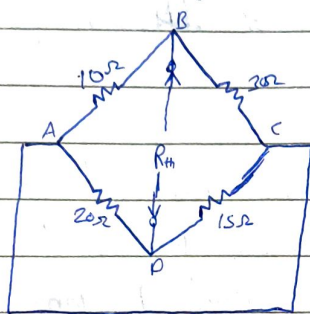
Voltage drop at point A to B = $2 - 0.5 = 1.5V$

30 Voltage drop at pt A to D = $2 - 1.14 = 0.86V$

$$\therefore V_{th} = 1.5 - 0.86$$

$$= 0.64V$$

Step ② :- To determine resistance R_{th} looking between terminals B & D, the source is replaced by its internal resistance. (zero in this case)

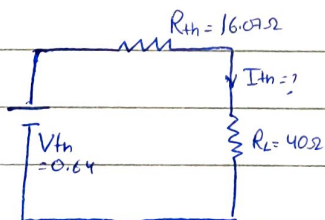


$$R_{th} = \frac{10 \times 30}{10 + 30} + \frac{20 \times 15}{20 + 15}$$

$$R_{th} = 7.5 + 8.57$$

$$[R_{th} = 16.07 \Omega]$$

Step ③ To determine current at 40Ω resistance We redraw the circuit to thevenin equivalent circuit.



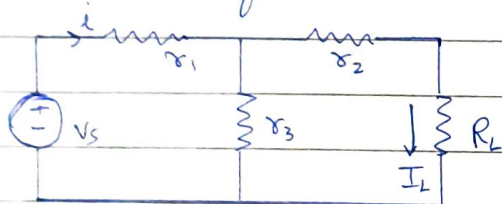
$$I_{th} = \frac{V_{th}}{R_{th} + R_L} = \frac{0.64}{16.07 + 40}$$

$$I_{th} = 0.0114 \text{ Amp or } 11.4 \text{ mA}$$

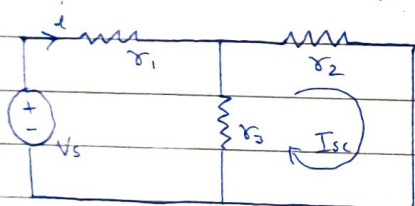
24 Norton's theorem

Norton's theorem states that "The current flowing through a resistance connected across any two terminals of network can be determined by replacing the whole network by an equivalent circuit of a current source having a current output of I_n in parallel with a resistance R_n "

Where, I_n is the short circuit current supplied by the source called Norton's Current. & R_n is the equivalent resistance of the network called Norton's resistance.



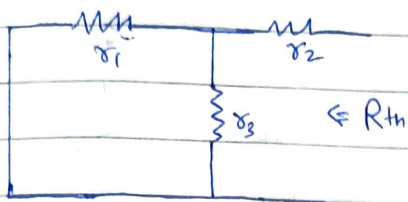
Step ①:- In order to find the current through R_L by Norton's theorem, let us replace R_L by short circuit.



$$I = \frac{V_s}{r_1 + \frac{r_2 r_3}{r_2 + r_3}}$$

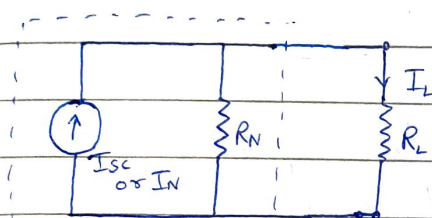
$$I_{sc} = I \times \frac{r_3}{r_3 + r_2}$$

Step ②:- To determine Norton's equivalent resistance the short circuit is removed. & the independent source is deactivated as done in Thevenin.



$$R_N = \frac{r_1 r_3}{r_1 + r_3} + r_2$$

Step ③:- To determine current in load resistance R_L as per Norton's theorem, the equivalent source circuit would contain current source in parallel to the internal resistance the current source being the short circuited current across the shorted terminal of the load resistor.



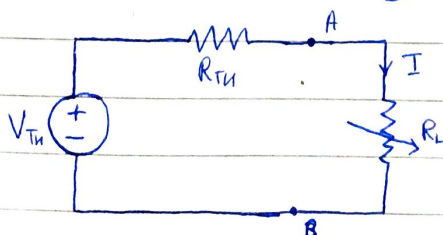
Norton equivalent circuit

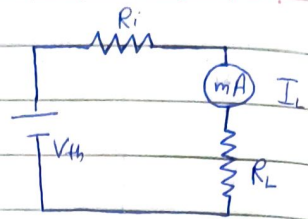
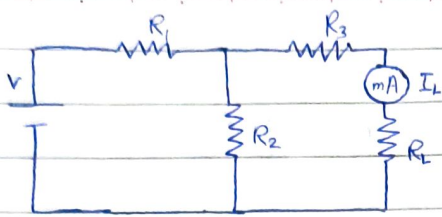
$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

25/10 Maximum power transfer theorem

The maximum power transfer theorem states that to achieve maximum delivery to a load, the load resistance must be equal to the source internal resistance, or thevenin's resistance (R_{th}).

For a DC circuit when the load resistance R_L equals the source resistance R_{th} , the power delivered to the load is maximized.





Load versus source resistance:

Maximum power is transfer when the load resistance R_L is equal to source resistance R_{th} ($R_L = R_{th}$).

Thevenin's Theorem:

The source network can be simplified to a single voltage source V_{th} in series with a single resistance R_{th} & the maximum power condition applies to this equivalent circuit.

[Formula:- The maximum power P_{max} delivered to the load is given by the formula.

$$P_{max} = V \cdot I_L$$

$$= I_L R_L \cdot I_L$$

$$= I_L^2 R_L$$

$$P_{max} = \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 \times R_L$$

$$P_{max} = \frac{V_{th}^2}{(2R_L)^2} \times R_L$$

$$(\because R_{th} = R_L)$$

$$P_{\max} = \frac{V_{th}^2}{4R_L} \times R_L$$

$$\left[P_{\max} = \frac{V_{th}^2}{4R_L} \right]$$

Step ① Find the Thevenin's equivalent:- Determine the thevenin voltage (V_{th}) & thevenin resistance (R_{th}) of the source network looking from the load's perspective

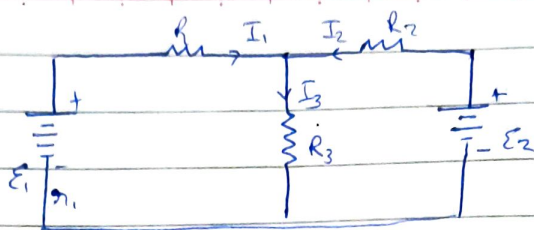
Step ② Match the resistances:- For maximum power the load resistance R_L must be equal to R_{th} .

Step ③ Calculate maximum power:- Once the resistances are matched use the formula:-

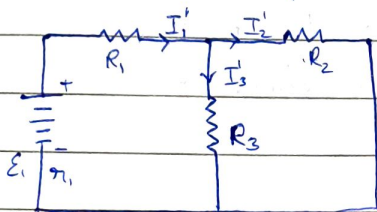
$$\left[P_{\max} = \frac{V_{th}^2}{4R_{th}} \right] \text{ to find max power delivered to the load.}$$

2.2) Super position theorem

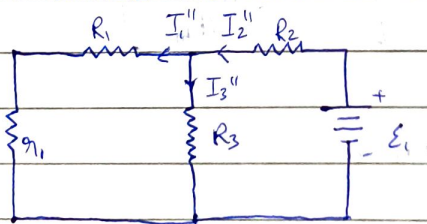
According to this theorem, If there are two or more than two sources of EMF acting simultaneously in a linear bilateral network, The current flowing through any section is the algebraic sum of all the currents which should flow in that section if each source of EMF were considered separately and all other sources are replaced, for the time being by their internal resistance.



Step ① :- Consider only one source ~~even~~ E_1 & replace the other source E_2 by its internal resistance. If internal resistance not given then it is taken as short circuit & determine currents in various section as I_1' , I_2' & I_3' respectively



Step ② :- Then consider the other source & replace the source E_1 by its internal resistance. Determine currents in various sections as I_1'' , I_2'' & I_3'' respectively



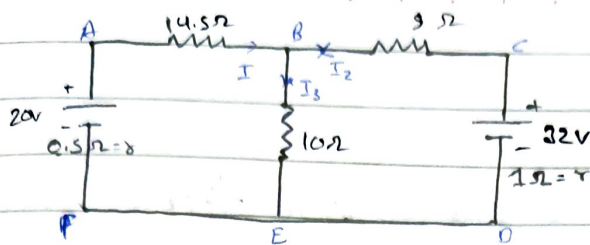
Step ③ :- Actual flow of current in various section

$$I_1 = I_1' - I_1''$$

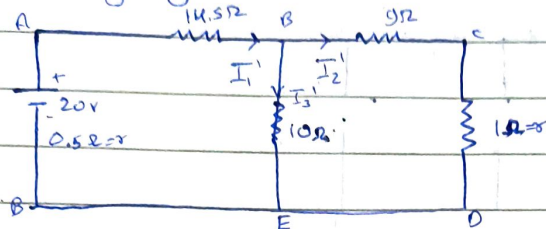
$$I_2 = I_2' - I_2''$$

$$I_3 = I_3' + I_3''$$

Q) Determine the branch current by superposition theorem in the network shown



⇒ Step: ① According to super position theorem replacing 32 volt battery by its internal resistance.



$$R_{20V} = 15 + \frac{10 \times 10}{10 + 10}$$

$$R_{20V} = 20\Omega$$

$$I_1' = \frac{V}{R_{20V}} = \frac{20}{20}$$

$$[I_1' = 1A]$$

Current in branch BCDE

$$I_2' = I_1' \times \frac{10}{10 + 10}$$

$$= 1 \times \frac{10}{20}$$

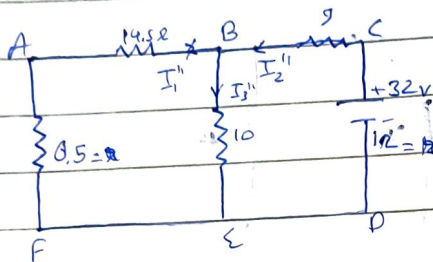
$$[I_2' = 0.5A]$$

Current in Branch BECD

$$I_3' = I_1' \times \frac{10}{10 + 10}$$

$$I_2' = 0.5 \text{ A}$$

Step ② According to superposition theorem replacing 20V battery by its internal resistance & now considering the source current 32V



$$R_{32V} = 10 + \frac{15 \times 10}{15 + 10}$$

$$R_{32} = 16$$

$$I_2'' = \frac{V}{R_{32}}$$

$$I_2'' = \frac{32}{16}$$

$$I_2'' = 2$$

Applying current divider rule in branch ABFE

$$I_1'' = I_2'' \times \frac{10}{15 + 10}$$

$$I_1'' = 2 \times \frac{10}{25}$$

$$[I_1'' = 0.8 \text{ A}]$$

Applying current divider rule in branch EBAF BEFA

$$I_3'' = I_2'' \times \frac{15}{10+15}$$

$$= 2 \times \frac{15}{25}$$

$$[I_3'' = 1.2 A]$$

Step ③ Actual flow of current in various branches is obtained by super position:-

$$I_1 = I_1' - I_1''$$

$$I_1 = 1 - 0.8$$

$$[I_1 = 0.2 A]$$

$$I_2 = I_2' - I_2''$$

$$= 0.5 - 2$$

$$[I_2 = -1.5 A]$$

$$I_3 = I_3' - I_3''$$

$$= 0.5 + 1.2$$

$$[I_3 = 1.7 A]$$

Unit-3

camlin

Date

Single phase AC circuit

★ J- operator:-

The J- operator is a mathematical symbol that is used to represent the complex numbers for ex. The J- operator is used in the form of $x+jy$ where x is a real number & y is a complex number.

$$j = \sqrt{-1}$$

$$\begin{aligned} j^2 &= j \cdot j \\ &= \sqrt{-1} \times \sqrt{-1} \\ &= (\sqrt{-1})^2 = -1 \end{aligned}$$

$$\begin{aligned} j^3 &= j^2 \cdot j \\ &= -1 \cdot j \\ &= -j \end{aligned} \quad , \quad \begin{aligned} j^4 &= j^2 \cdot j^2 \\ &= (-1) \cdot (-1) \\ &= 1 \end{aligned}$$

★ Rectangular form

$A = a + jb$ form is called rectangular form where,

a = is known as real component

b = is a imaginary component

★ Polar form:-

$A = r \angle \theta$ form is known as polar form where, r is the magnitude of the vector

$$[r = \sqrt{a^2 + b^2}]$$

and θ indicates the angle

$$\therefore \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

★ Rectangular to polar conversion:-

Q.1) $A = 3 + j4$

⇒ Given,

$$A = 3 + j4$$

$$a = 3$$

$$b = 4$$

We know that polar form
 $r \angle \theta$

$$r = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$

$$r = 5$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$= \tan^{-1} \left(\frac{4}{3} \right)$$

$$= 53.13^\circ$$

$$A = 3 + j4 = 5 \angle 53.13$$

Qn) $A = 16 + j12$

⇒ Given,

$$A = 16 + j12$$

$$a = 16$$

$$b = 12$$

We know that polar form
 $r \angle \theta$

$$r = \sqrt{a^2 + b^2}$$

$$= \sqrt{16^2 + 12^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400}$$

$$r = 20$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{12}{16}\right)$$

$$\theta = 36.86$$

$$A = 16 + j12$$

$$[A = 20 \angle 36.86] \text{ Answer}$$

★ Addition & subtraction for of vector quantity:-

if $A_1 = a_1 + jb_1$
& $A_2 = a_2 + jb_2$

$$A = A_1 + A_2 = a_1 + jb_1 + a_2 + jb_2$$

$$= (a_1 + a_2) + j(b_1 + b_2)$$

★ Polar form:

$$A = r \angle \theta$$

$$r = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$$

$$\theta = \tan^{-1} \left(\frac{b_1 + b_2}{a_1 + a_2} \right)$$

(Q) Add the following vector given in vector form:-

$$A = 16 + 12j$$

$$B = -6 + 10.4j$$

⇒ Soln Given:-

$$A = 16 + 12j$$

$$B = -6 + 10.4j$$

$$A + B = 16 + 12j + \{(-6) + 10.4j\}$$

$$= 10 + 22.4j$$

$$[\text{Polar form} = r \angle \theta = 24.53 \angle 65.94] \text{ Answer}$$

Qn) Subtracting the following vector:-

$$A = 30 + j52$$

$$B = -39.5 - j14.36$$

→ Given ;

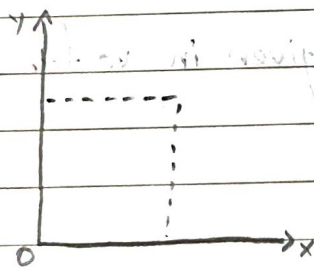
$$A = 30 + j52$$

$$B = -39.5 - j14.36$$

$$\begin{aligned} A - B &= 30 + j52 - (-39.5 - j14.36) \\ &= 30 + j52 + 39.5 + j14.36 \\ &= 69.5 + 66.36j \end{aligned}$$

$$[\text{Polar form} = r \angle \theta = 96.09 \angle 43.67]$$

★ Polar to rectangular form :-



X component of ~~small~~ r is $r \cos \theta$ & Y component of r is $r \sin \theta$, hence, we can represent the vector r to the form of $r(\cos \theta + j \sin \theta)$. This is equivalent to the rectangular form $a + jb$ hence, $a = r \cos \theta$ & $b = r \sin \theta$

Qn) Convert polar to rectangular form

1) $20 \angle -120^\circ$

Given,

$$r = 20$$

$$\theta = -120^\circ$$

$$a = r \cos \theta$$

$$a = 20 \cos(-120^\circ)$$

$$a = -10$$

$$b = r \sin \theta$$

$$b = 20 \sin(-120^\circ)$$

$$b = -17.32$$

Hence, rectangular form $[a + jb = -10 - j17.32]$ - Answer

★ Multiplication & division of vector quantity:-

$$\Rightarrow \text{If } A = r_1 \angle \theta_1$$

$$\& B = r_2 \angle \theta_2$$

Multiplication of A & B

$$A \times B = r_1 r_2 \angle \theta_1 + \theta_2$$

Division of A & B

$$\frac{A}{B} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

★ Conjugate form:-

$$A = Z \angle \theta$$

$$A^* = Z \angle -\theta$$

Qn) If $A = 5 + j5$

$$B = 50 \angle 40$$

$$\& C = 40 + j10$$

Then find:- 1) $\frac{AB}{C}$ 2) $\frac{BC}{A}$

⇒ Given,

$$A = 5 + j5 = 7.07 \angle 45$$

$$B = 50 \angle 40$$

$$C = 40 + j10 = 41.23 \angle 14.03 \quad 10.77 \angle 68.19$$

$$1) \frac{AB}{C} = \frac{7.07 \angle 45 \times 50 \angle 40}{10.77 \angle 68.19}$$

$$[= 31.62 + 9.55j = 33.03 \angle 16.81] \text{ Answer}$$

$$2) \frac{BC}{A} = \frac{50 \angle 40 \times 10.77 \angle 68.19}{7.07 \angle 45}$$

$$= \frac{50 \times 10.77 \angle 40 + 68.19}{7.07 \angle 45}$$

$$= 34.13 + 67.53j$$

$$[\text{in polar form} \Rightarrow 76.18 \angle 63.19] \text{ Answer}$$

Q2 If $Z_1 = 16 \angle -30^\circ$

& $Z_2 = 5 \angle 30^\circ$ then find:- 1) $Z_1 + Z_2$ 2) $Z_1 - Z_2$

3) $Z_1 \times Z_2$ 4) Z_1 / Z_2

5) $Z_1^* + Z_2^*$

⇒ Given:-

$$Z_1 = 16 \angle -30^\circ$$

$$Z_2 = 5 \angle 30^\circ$$

1) $Z_1 + Z_2 = 16 \angle -30^\circ + 5 \angle 30^\circ$

=

$$= 18.18 - 5.5j$$

In polar form = $19 \angle -16.82^\circ$

2) $Z_1 - Z_2 =$

$$= 16 \angle -30^\circ + 5 \angle 30^\circ$$

$$= 9.52 - 10.5j$$

In polar form = $14.17 \angle -47.7^\circ$

3) $Z_1 \times Z_2 = 16 \times 5 + \angle -30^\circ + 30^\circ$

$$= 80 \angle 0^\circ$$

4) $\frac{Z_1}{Z_2} = \frac{16 \angle -30^\circ}{5 \angle 30^\circ} = \frac{16}{5} \angle -30^\circ - 30^\circ$

$$\left[\frac{Z_1}{Z_2} = 3.2 \angle -60^\circ \right]$$

Answer

5) $Z_1^* + Z_2^*$

$$Z_1^* = 60 \angle 30^\circ$$

$$Z_2^* = 5 \angle -30^\circ$$

$$Z^* \cdot Z^* = 16 \angle 30 + 5 \angle -30$$

$$= 18.18 + 5.5j$$

[In polar form = $19 \angle 16.82$]

Qn IF,

$$Z_1 = 10 \angle 30$$

$$Z_2 = 15 \angle 60$$

then find:-

1) $Z_1 + Z_2$

4) $Z_1 \times Z_2$

2) $Z_1 - Z_2$

5) $Z_1^* + Z_2^*$

3) Z_1 / Z_2

⇒ Given,

$$Z_1 = 10 \angle 30$$

$$Z_2 = 15 \angle 60$$

$$\begin{aligned} 1) Z_1 + Z_2 &= 10 \angle 30 + 15 \angle 60 \\ &= 16.16 + 17.99j \end{aligned}$$

[In polar = $24.18 \angle 48.06$] Answer

$$\begin{aligned} 2) Z_1 - Z_2 &= 10 \angle 30 - 15 \angle 60 \\ &= 1.16 - 7.99j \end{aligned}$$

[In polar = $8.07 \angle -81.73$] Answer

$$3) Z_1 / Z_2 = \frac{10 \angle 30}{15 \angle 60} = \frac{10}{15} \angle 30 - 60$$

$$= \quad \quad = [0.57 + 0.33j]$$

[In polar form = $0.66 \angle -30$]

$$\begin{aligned} 4) Z_1 \times Z_2 &= 10 \angle 30 \times 15 \angle 60 \\ &= 10 \times 15 \angle 30 + 60 \\ &= 150 \angle 90 \\ &= 150j \end{aligned}$$

$$\begin{aligned} 5) Z_1^* + Z_2^* &= 10 \angle -30 + 15 \angle -60 \\ &= 16.16 - 17.99j \end{aligned}$$

$$\text{In polar} = 24.18 \angle -48.06$$

Sinusoidal steady state Analysis:-

Sinusoidal $\begin{cases} \rightarrow \text{sine} \\ \rightarrow \text{cosine} \end{cases}$

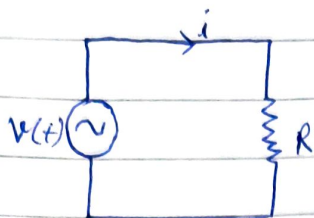
$$V(t) = V_m \sin \omega t$$

$V_m \rightarrow$ Max. amplitude

$\omega \rightarrow$ Angular freq

$\omega t \rightarrow$ Argument

1) Resistor:-



$$v(t) = V_m \sin \omega t$$

$$i(t) = \frac{v(t)}{R}$$

$$= V_m \sin \omega t$$

$$[i(t) = I_m \sin \omega t]$$

$$\vec{I} \rightarrow \vec{V}$$

$$* P_{av} = \frac{1}{T} \int_0^T P(t) dt$$

$$= \frac{1}{T} \int_0^T v(t) \cdot i(t) dt$$

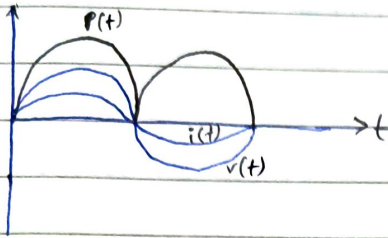
$$= \frac{1}{T} \int_0^T (V_m \sin \omega t) (I_m \sin \omega t) dt$$

$$= \frac{1}{T} \int_0^T V_m I_m \sin^2 \omega t dt$$

$$P_{av} = \frac{V_m I_m}{2T} \int_0^T (1 - \cos 2\omega t) dt$$

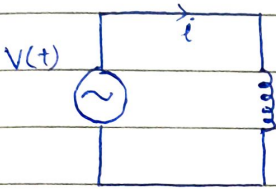
$$P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$= V_{rms} \cdot I_{rms}$$



$$f_p = 2f_v / f_i$$

2) Inductor:-



$$i(t) = I_m \sin \omega t$$

$$v(t) = L \frac{di}{dt}$$

$$= L \frac{d}{dt} (I_m \sin \omega t)$$

$$= \omega L I_m \cos \omega t$$

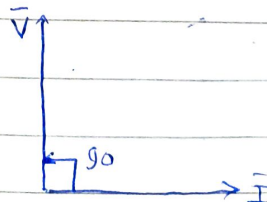
$$= \omega L I_m \sin(\omega t + 90^\circ)$$

$$v(t) = j\omega L I_m \sin \omega t$$

$$v(t) = jX_L I_m \sin \omega t$$

$$X_L = \omega L$$

Inductive reactance



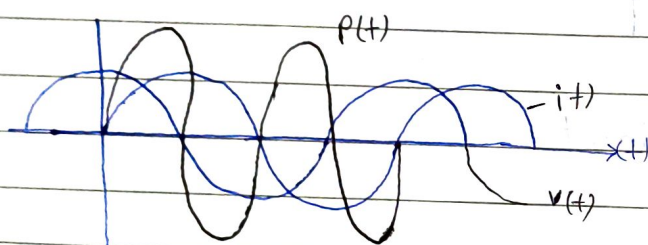
$$P_{av} = \frac{1}{T} \int_0^T P(t) dt$$

$$= \frac{1}{T} \int_0^T v(t) \cdot i(t) dt$$

$$= \frac{1}{T} \int_0^T (V_m \cos \omega t) (I_m \sin \omega t) dt$$

$$P_{av} = \frac{V_m I_m}{2T} \int_0^T \sin 2\omega t dt$$

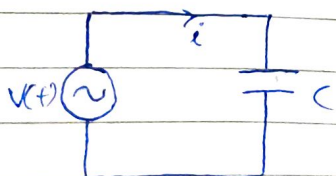
$$[P_{av} = 0]$$



$$f_p = 2f_v \text{ or } 2f_i$$

- * In the pre half cycle of the power inductor takes energy from the source & in the negative half cycle of power inductor delivers energy to the source

③ Capacitor



$$V(t) = V_m \sin \omega t$$

$$i(t) = \frac{C dv}{dt}$$

$$= C \frac{d(V_m \sin \omega t)}{dt}$$

$$i(t) = \omega C V_m \cos \omega t$$

$$i(t) = \omega C V_m \sin(\omega t + 90)$$

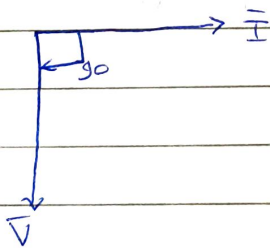
$$i(t) = j\omega C V_m \sin \omega t$$

$$V(t) = \frac{i(t)}{j\omega C}$$

$$V(t) = -jX_C \cdot i(t)$$

$$X_C = \frac{1}{\omega C}$$

X_C = capacitive reactance.



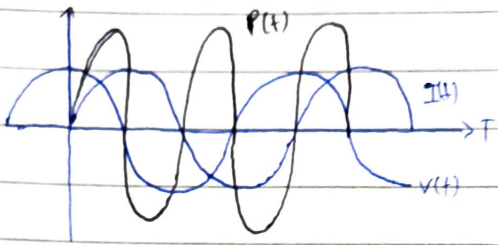
$$P_{av} = \frac{1}{T} \int_0^T P(t) dt$$

$$= \frac{1}{T} \int_0^T v(t) \cdot i(t) dt$$

$$= \frac{1}{T} \int_0^T (I_m \cos \omega t) \cdot (V_m \sin \omega t) dt$$

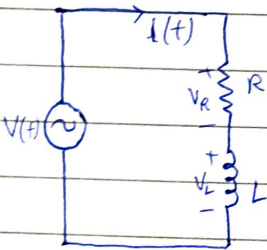
$$= \frac{V_m I_m}{2T} \int_0^T \sin 2\omega t dt$$

$$[P_{av} = 0]$$



$$f_p = 2f_v \text{ or } 2f_i$$

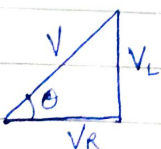
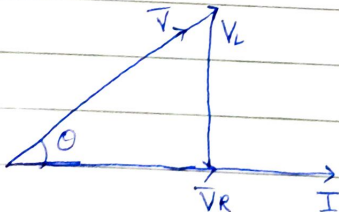
④ Series R-L circuit.



$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$\bar{I}_Z = \bar{I}_R + j\bar{I}_{XL}$$

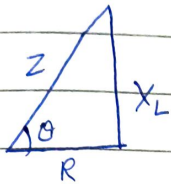
$$Z = R + jX_L$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\theta = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

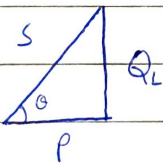
$$\cos \theta = \frac{V_R}{V} \text{ (lagg)}$$



$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\cos \theta = \frac{R}{Z} \text{ (lagg)}$$



$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1} \left(\frac{Q_L}{P} \right)$$

$$\cos \theta = \frac{P}{S} \text{ (lagg)}$$

- Power factor angle indicates the position of the current phasor with respect to the voltage phasor
- While defining the power factor for any circuit voltage

phasor with respect to is taken as a reference because in the real time s/s all the load are connected in parallel.

- Power factor tells us how much the power is utilized.

$$P_{av} = \frac{1}{T} \int_0^T P(t) dt$$

$$= \frac{1}{T} \int_0^T v(t) i(t) dt$$

$$= \frac{1}{T} \int_0^T V_m \sin(\omega t + \theta) \cdot I_m \sin \omega t dt$$

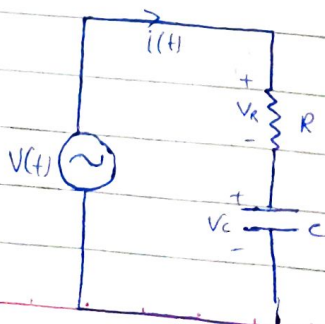
$$= \frac{V_m I_m}{2T} \int_0^T (\cos \theta - \cos(2\omega t + \theta)) dt$$

$$P_{av} = \frac{V_m I_m}{2} \cos \theta$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta$$

$$P_{av} = V_{rms} \cdot I_{rms} \cos \theta$$

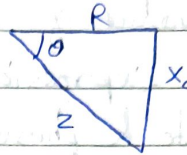
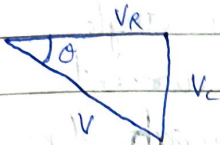
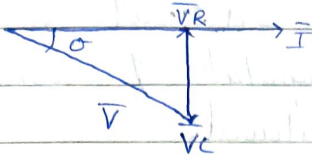
⑤ Series R-C circuit



$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$\bar{I}_Z = \bar{I}_R - jX_C \bar{I}$$

$$Z = R - jX_C$$



$$V = \sqrt{V_R^2 + V_C^2}$$

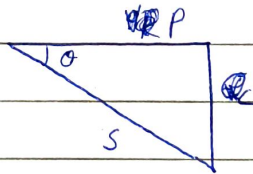
$$Z = \sqrt{R^2 + X_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-V_C}{V_R} \right)$$

$$\theta = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

$$\cos \theta = \frac{V_R}{V} \text{ (lead)}$$

$$\cos \theta = \frac{R}{Z} \text{ (lead)}$$



Power Triangle

$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-Q_C}{P} \right)$$

$$\cos \theta = \frac{P}{S} \text{ (lead)}$$

Ex:- If input angle (-ve) then power factor (+ve)

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{\bar{V}}{|Z| \angle \theta}$$

$$\bar{I} = \frac{\bar{V}}{|Z|} \angle -\theta$$

Qn) 50 Hz sinusoidal voltage = $311 \sin \omega t$ is applied to a R-L series circuit if magnitude of resistance is 5Ω & inductance is 0.02 H calculate

- 1) RMS value of current & its phase angle
- 2) Obtain expression of instantaneous current
- 3) Magnitude & phase of voltage drop in each element.

$\Rightarrow$ Given that,

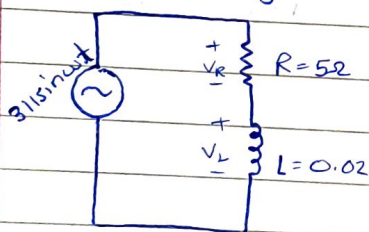
$$f = 50 \text{ Hz}$$

$$V(t) = 311 \sin \omega t$$

$$R = 5 \Omega$$

$$L = 0.02 \text{ H}$$

Circuit diagram



$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} \quad \text{or} \quad \frac{V_{\text{rms}}}{Z}$$

$$I_m = \frac{V_m}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$X_L = 2\pi fL$$

$$X_L = 6.28 \Omega$$

$$Z = \sqrt{5^2 + (6.28)^2}$$

$$Z = 8.02 \Omega$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{6.28}{5} \right)$$

$$\phi = 51.47$$

$$\textcircled{1} I_{rms} = \frac{V_{rms}}{Z}$$

$$Z = 8.02 \angle 51.47$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 220V$$

$$I_{rms} = \frac{220}{8.02 \angle 51.47}$$

$$I_{rms} = 27.4 \angle -51.47$$

2) Expression for instantaneous current

$$i = I_m \sin(\omega t - \phi)$$

$$i = 38.74 \sin(\omega t - 51.47)$$

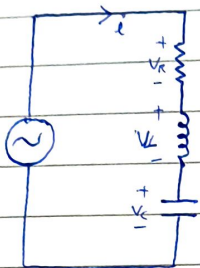
$$\textcircled{3} V_R = IR = 137 \angle -51.47$$

$$V_L = jIX_L$$

$$= j \times 27.4 \angle 51.47^\circ \times 6.28$$

$$= 172.07 \angle 38.53^\circ \quad \text{Answer}$$

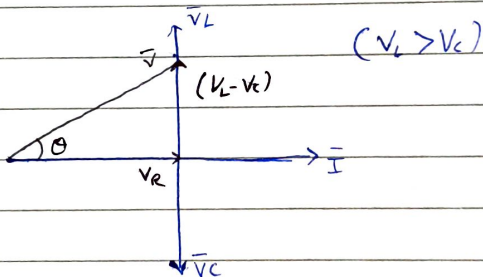
★ Series R-L-C circuit.



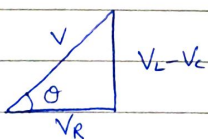
$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

$$\bar{I}Z = \bar{I}R + jX_L\bar{I} - jX_C\bar{I}$$

$$Z = R + j(X_L - X_C)$$



Voltage triangle:

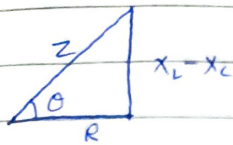


$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

$$\cos \theta = \frac{V_R}{V}$$

Impedance triangle

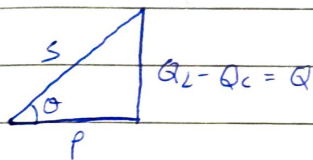


$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\cos \theta = \frac{R}{Z}$$

Power triangle

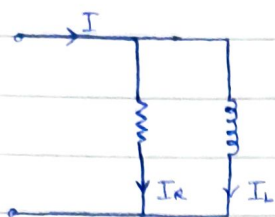


$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1} \left(\frac{Q}{P} \right)$$

$$\cos \theta = \frac{P}{S}$$

Power (Avg) is same as written in R-L-series circuit.

★ Parallel R-L circuit

$$\bar{I} = \bar{I}_R + \bar{I}_L$$

$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{jX_L}$$

$$\boxed{Y = G - jB_L}$$

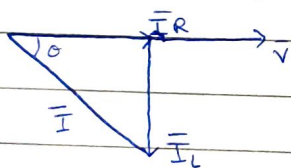
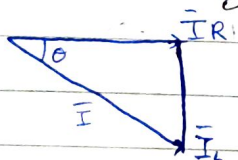
where,

Y = Admittance

G = Conductance

B_L = Susceptance

Phasor diagram

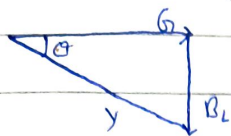
★ Current triangle

$$\bar{I} = \sqrt{\bar{I}_R^2 + \bar{I}_L^2}$$

$$\theta = \tan^{-1} \left(\frac{I_L}{I_R} \right)$$

$$\cos \theta = \frac{I_R}{I} \text{ (lagg)}$$

2) Admittance triangle

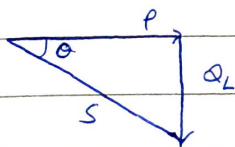


$$Y = \sqrt{G^2 + (B_L)^2}$$

$$\theta = \tan^{-1} \left(-\frac{B_L}{G} \right)$$

$$\cos \theta = \frac{G}{Y}$$

3) Power triangle



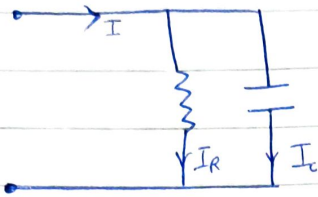
$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1} \left(-\frac{Q_L}{P} \right)$$

$$\cos \theta = \frac{P}{S} \text{ (lagg)}$$

* $\theta \Rightarrow$ minus (-)

$\therefore P \text{ :- lagg } (\because \theta \rightarrow \text{admittance angle})$

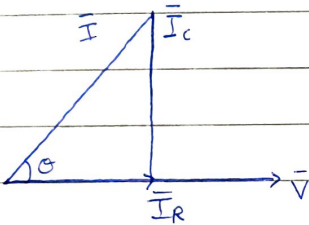
★ Parallel R-C circuit

$$\bar{I} = \bar{I}_R + \bar{I}_C$$

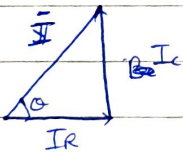
$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{-jX_C}$$

$$[Y = G + jB_C]$$

Phasor diagram



i) Current triangle

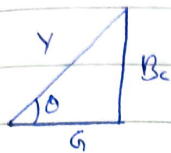


$$I = \sqrt{I_R^2 + I_C^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C}{I_R} \right)$$

$$\cos \theta = \frac{I_R}{I} \text{ (lead)}$$

Admittance triangle

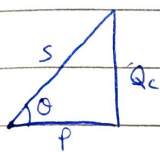


$$Y = \sqrt{G^2 + B_c^2}$$

$$\theta = \tan^{-1} \left(\frac{B_c}{G} \right)$$

$$\cos \theta = \frac{G}{Y} \text{ (lead)}$$

Power triangle

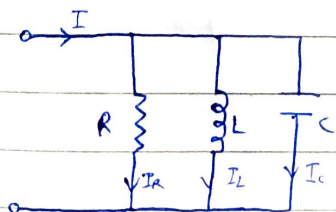


$$S = \sqrt{P^2 + Q_c^2}$$

$$\theta = \tan^{-1} \left(\frac{Q_c}{P} \right)$$

$$\cos \theta = \frac{P}{S} \text{ (lead)}$$

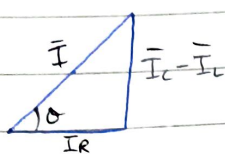
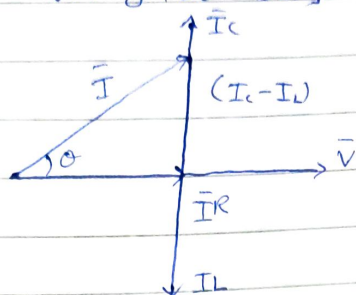
Parallel R-L-C circuit



$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{jX_L} + \frac{\bar{V}}{-jX_C} \Rightarrow Y$$

$$[Y = G + j(B_C - B_L)]$$



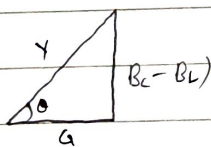
Current triangle

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

$$\cos \theta = \frac{I_R}{I}$$

Admittance triangle :-

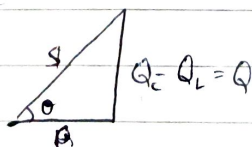


$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\theta = \tan^{-1} \left(\frac{B_C - B_L}{G} \right)$$

$$\cos \theta = \frac{G}{Y}$$

Power triangle :-



$$\theta = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\cos \theta = \frac{P}{S}$$

Basic operations:-

$$\begin{aligned} Z &= x + jy \\ &= r_1 e^{j\theta} \\ &= r_1 \angle \theta \end{aligned}$$

$$r_1 = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$Z_1 = x_1 + jy_1 = r_1 \angle \theta_1$$

$$Z_2 = x_2 + jy_2 = r_2 \angle \theta_2$$

1) Addition:-

$$Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

2) Subtraction:-

$$Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

3) Multiplication:-

$$Z_1 Z_2 = r_1 r_2 \angle \theta_1 + \theta_2$$

4) Division:-

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

5) Reciprocal:-

$$\frac{1}{z} = \frac{1}{r_1} \angle -\theta$$

6) Square root:-

$$\sqrt{z} = \sqrt{r_1} \angle \frac{\theta}{2}$$

$$\begin{cases} z^n = (r_1 e^{j\theta})^n \\ = r_1^n e^{j(n\theta)} \\ = r_1^n \angle n\theta \end{cases}$$

7) Conjugate:-

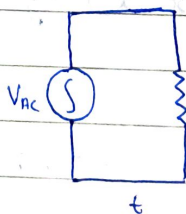
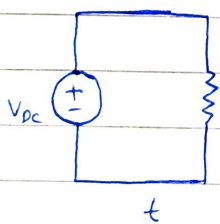
$$z^* = x - jy = r_1 \angle -\theta$$

{Notes}

★ RMS Value:-

1) RMS value is defined based on the heating effect of the waveform.

2) The voltage at which heat dissipation in AC circuit is equal to the heat dissipated in DC circuit is called as a RMS ~~power~~ value provided to both AC & DC circuits have equal value of Resistance $\times$ are operated for the same time interval.



$$W_{\text{heat}} \rightarrow W_{ac} = W_{dc}$$

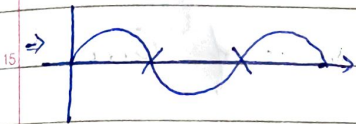
$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V^2 dt}$$

★ Average value :- Average value is defined based on the charge transfer in the circuit.

- The voltage at which charge transfer in an AC circuit is equal to the charge transfer in DC circuit is called as the average value provided both AC & DC circuit have equal value of resistance & are operated for same time interval.

★ Unsymmetrical :-

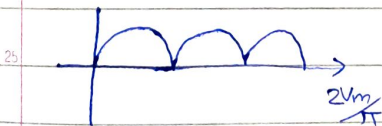
$$V_{av} = \frac{1}{T} \int_0^T v dt$$



★ Symmetrical

$$V_{av} = 0 \Rightarrow \text{full cycle}$$

$$V_{av} = \frac{1}{T/2} \int_0^{T/2} v dt \Rightarrow \text{Half cycle}$$



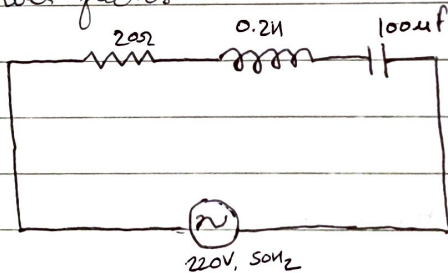
★ Form factor:- It is the ratio of the RMS value of the waveform to the average value of the wave-form.

$$\left[\text{form factor (FF)} = \frac{V_{\text{rms}}}{V_{\text{avg}}} \right]$$

★ Peak factor:- It is the ratio of the max value of the waveform to the RMS value of the waveform.

$$\left[\text{peak factor (PF)} = \frac{V_m}{V_{\text{rms}}} \right]$$

Qn) For the given circuit find the value of Current impedance & power factor



Solution:-

Given that:-

$$R = 20\Omega$$

$$L = 0.2H$$

$$C = 100\mu F$$

$$f = 50Hz$$

$$V_{\text{rms}} = 220V$$

We know that

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = 2\pi fL$$

$$X_L = 2\pi \times 50 \times 0.2$$

$$X_L = 62.83$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$X_C = ~~31.8~~ 31.83$$

$$Z = 36.89$$

By Ohms law = $I = \frac{V}{Z}$

$$= \frac{220}{36.89}$$

$$= 5.96 A$$

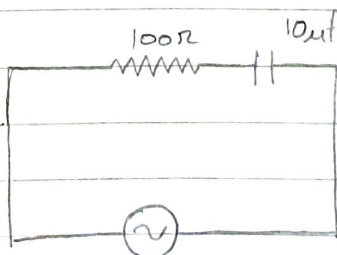
Power factor = $\cos \theta = \left(\frac{R}{Z} \right)$

$$= 0.54$$

Answer

Power factor =

Qn) In a series RC circuit the source current is given by $i = 10 \sin(500t)$ where $r = 100 \Omega$ & $C = 10 \mu F$ find the applied voltage.



⇒ Given,

$$i = 10 \sin(5000t)$$

$$R = 100 \Omega$$

$$C = 10 \mu\text{f}$$

$$V = V_m \sin(\omega t \pm \phi)$$

$$V_{\text{rms}} = I_{\text{rms}} \times Z \quad \text{by Ohm's law}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = 20 \Omega$$

$$Z = \sqrt{100^2 + 20^2}$$

$$\theta = \tan^{-1} \left(\frac{X_C}{R} \right)$$

$$Z = 101.98 \Omega$$

$$\phi = 11.30^\circ$$

$$\begin{aligned} \therefore V_m &= I_m \times Z \\ &= 10 \times 101.98 \\ &= 1019.8 \end{aligned}$$

$$\begin{aligned} I_{\text{rms}} &= \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} \\ &= 7.07 \end{aligned}$$

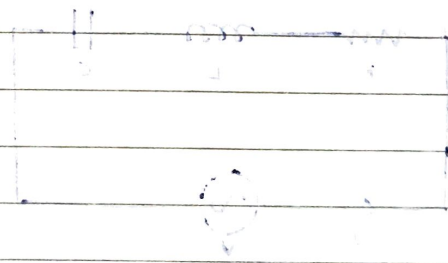
$$\begin{aligned} V_{\text{rms}} &= I_{\text{rms}} \times Z \\ &= 7.07 \times 101.98 \end{aligned}$$

$$V_{\text{rms}} = 720.9 \text{ V}$$

$$\begin{aligned} V_m &= V_{\text{rms}} \sqrt{2} \\ &= 720.9 \times \sqrt{2} \\ V_m &= 1019.5 \text{ V} \end{aligned}$$

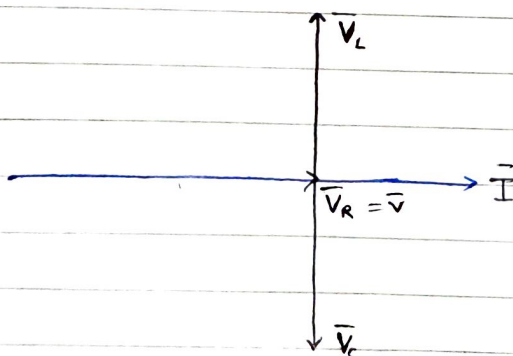
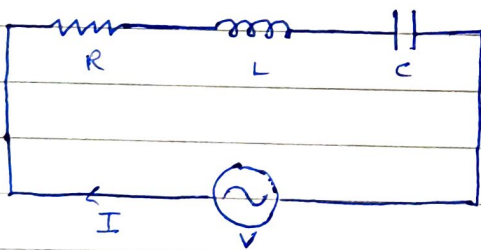
$$V = V_m \sin(\omega t - \phi) \quad \text{for } X_C \text{ circuit}$$

$$[V = 1019.5 \sin(5000t - 11.30^\circ)] \quad \text{Answer}$$



Series & Parallel resonance★ Resonance:-

- For occurrence of resonance in any s/c two energy -es are required and in an e-cal s/c. This two energy are provided by inductor & capacitor in the form of magnetic field & Electric field energy respectively.
- The circuit is said to be at resonance when the source voltage & source current are in phase.
- The resonant frequency indicates the rate at which energy transformation takes place between the inductor and the capacitor.

★ Series RLC circuit:-

$$|V_L| = |V_C|$$

$$IX_L = IX_C$$

$$[X_L = X_C]$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

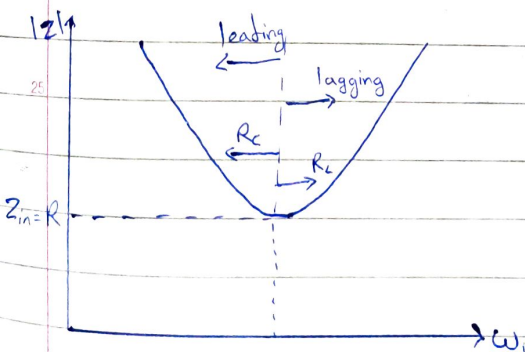
$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$Z = R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



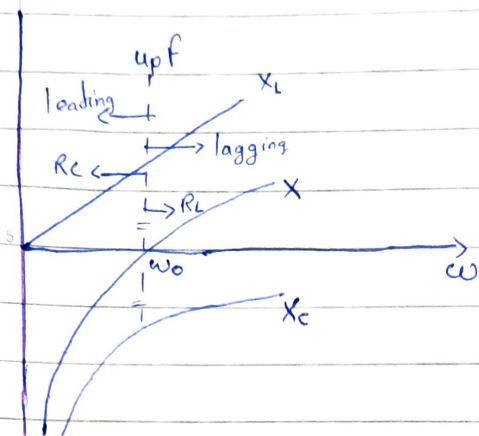
initially $\omega_c = \infty$
at instant $\frac{1}{\omega_c} = \omega_L$

$$|Z| = R$$

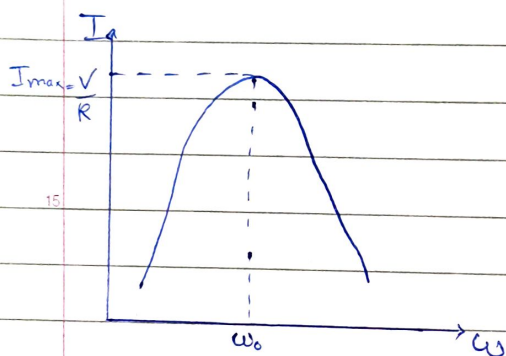
& after that

X_L will be increased

$$\begin{aligned} 1) X &= X_L - X_C \\ &= \omega L - \frac{1}{\omega C} \end{aligned}$$



③ I vs ω



$$I = \frac{V}{Z}$$

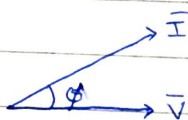
★ For $\omega < \omega_0$

$$Z = R - jX$$

↓

R_c

★ $P_f \rightarrow$ leading



★ At $\omega = \omega_0$

$$Z = R$$

$P_f = \text{unity}$

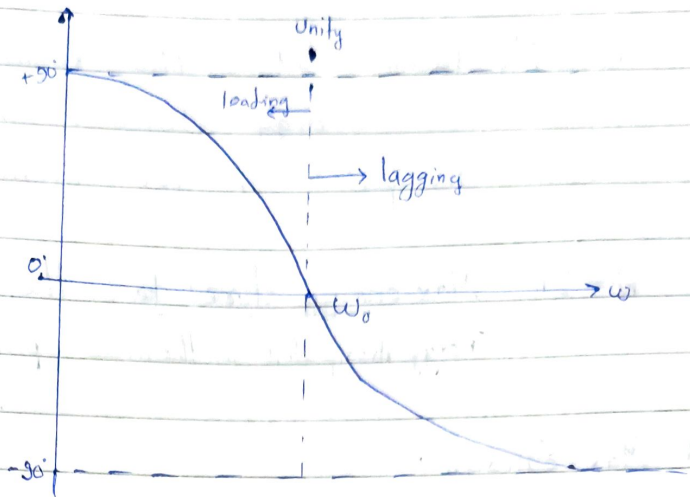


for $\omega > \omega_0$

$$Z = R + jX$$

↓
RL

pf $\rightarrow$ lagging



- 1) for $\omega < \omega_0$ the capacitive reactance is higher than the inductive reactance i.e.
- $$|X_L| < |X_C|$$

& hence the net reactance is capacitive in nature, the current I leads the applied voltage V & therefore the power factor is leading in nature.

- 2) At $\omega = \omega_0$ both the reactances are equal in magnitude i.e. $|X_L| = |X_C|$ & hence the impedance is minimum ($Z = R$). The current I is in phase with the applied voltage V & hence the p.f. is unity.

- 3) For $\omega > \omega_0$ inductive reactance is higher than the capacitive reactance i.e. $|X_L| > |X_C|$ and hence the net reactance is inductive in nature.

The current I lags the applied voltage V & hence the p.f. is lagging in nature.

★ Quality factor:- It is the ratio of the maximum energy stored in the circuit to the energy dissipated in the circuit per cycle of oscillation.

$$Q = 2\pi \frac{\text{Max energy stored in the circuit}}{\text{Energy dissipated in the circuit per cycle of oscillation}}$$

★ Max. Energy:-

$$W_m = \frac{1}{2} L i^2 + \frac{1}{2} (V_c)^2$$

$$i = I_m \sin \omega t$$

$$V_c = \frac{1}{C} \int i dt$$

$$= \frac{1}{C} \int I_m \sin \omega t dt$$

$$= -\frac{1}{\omega C} I_m \cos \omega t$$

$$W_m = \frac{1}{2} L (I_m \sin \omega t)^2 + \frac{1}{2} C \left(-\frac{I_m}{\omega C} \cos \omega t \right)^2$$

$$= \frac{1}{2} L I_m^2 \sin^2 \omega t + \frac{1}{2} \frac{I_m^2}{\omega^2 C} \cos^2 \omega t$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$W_m = \frac{1}{2} L I_m^2 \sin^2 \omega t + \frac{1}{2} L I_m^2 \cos^2 \omega t$$

$$= \frac{1}{2} L I_m^2 [\sin^2 \omega t + \cos^2 \omega t]$$

$$W_m = \frac{1}{2} L I_m^2$$

$$\therefore Q = 2\pi \times \frac{\frac{1}{2} L I_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2_{R, I}}$$

$$Q = 2\pi \frac{1}{T} \times \frac{L}{R}$$

$$Q = 2\pi f \frac{L}{R}$$

$$Q = \frac{\omega L}{R}$$

$$\star Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{X_L}{R} = \frac{X_C}{R}$$

$$Q = \frac{1}{WR_C}$$

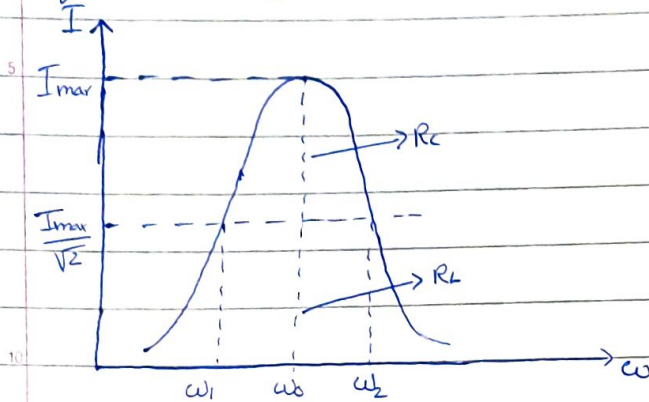
$$Q = \frac{I X_L}{I R} = \frac{V_L}{V_R} = \frac{V_C}{V_R}$$

$$Q = \frac{\text{Reactive component of voltage}}{\text{Active component of voltage}}$$

$\star$ Bandwidth:

Band width represents the range of frequencies for

which the power level in the signal is at least half of maximum power.



* At $\omega = \omega_0$, $I = I_{\max}$, $Z = R$

* At $\omega = \omega_1, \omega_2$, $I = \frac{I_{\max}}{\sqrt{2}}$, $Z = \sqrt{2}R$

$$\sqrt{R^2 + X^2} = \sqrt{2}R$$

$$R^2 + X^2 = 2R^2$$

$$X^2 = R^2$$

$$[X = \pm R]$$

* At $\omega = \omega_1$, $\therefore X = -R$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{--- ①}$$

* At $\omega = \omega_2$; $X = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- ②}$$

Adding eqn ① & ②

$$(\omega_1 + \omega_2) L - \frac{1}{C} \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) = 0$$

$$(\omega_1 + \omega_2) L - \frac{1}{C} \left[\frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \right] = 0$$

$$L = \frac{1}{C \omega_1 \omega_2} \Rightarrow \omega_1 \omega_2 = \frac{1}{LC}$$

$$\star 10 \quad \left[\begin{array}{l} \omega_0^2 = \omega_1 \omega_2 \\ \omega_0 = \sqrt{\omega_1 \omega_2} \end{array} \right]$$

$\star$ On subtracting eqn (2) from eqn (1)

$$(\omega_2 - \omega_1) L + \frac{1}{C} \left[\frac{1}{\omega_1} - \frac{1}{\omega_2} \right] = 2R$$

$$(\omega_2 - \omega_1) L + \frac{1}{C} \left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right] = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{C \omega_1 \omega_2} \right] = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{C \cdot \frac{1}{LC}} \right] = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{\frac{1}{L}} \right] = 2R$$

$$\omega_2 - \omega_1 = \frac{2R}{2L}$$

$$\left[\text{B.W} = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec} \right]$$

★ At $\omega = \omega_1$,

$$Z = R + jX = R - jR$$

$$\text{impedance angle } \theta = -\tan^{-1} \frac{R}{R}$$

$$= -45^\circ$$

$$\cos \theta = \frac{1}{\sqrt{2}} = 0.707 \text{ (leading)}$$

{ Impedance opposite the }
Pg

★ At $\omega = \omega_2$

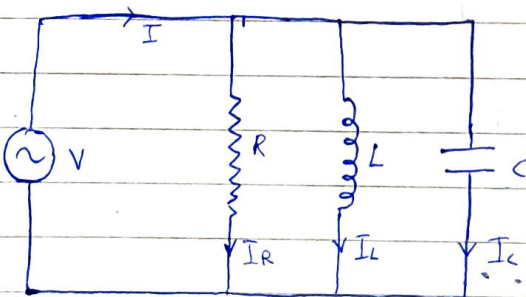
$$Z = R + jX = R + jR$$

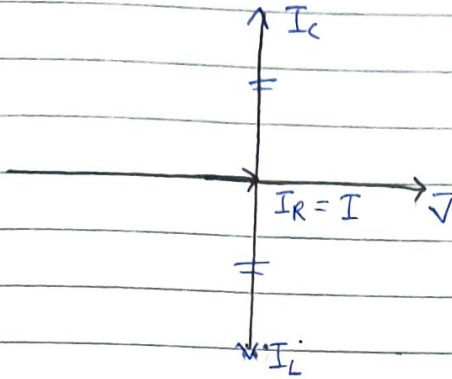
$$\text{impedance angle } \theta = \tan^{-1} \frac{R}{R}$$

$$= 45^\circ$$

$$\cos = \frac{1}{\sqrt{2}} = 0.707 \text{ (lagging)}$$

★ Parallel resonance :-





$$|I_L| = |I_c|$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$[B_L = B_C]$$

$$\star \frac{1}{\omega_0 L} = \omega_0 C$$

$$\omega^2 = \frac{1}{LC}$$

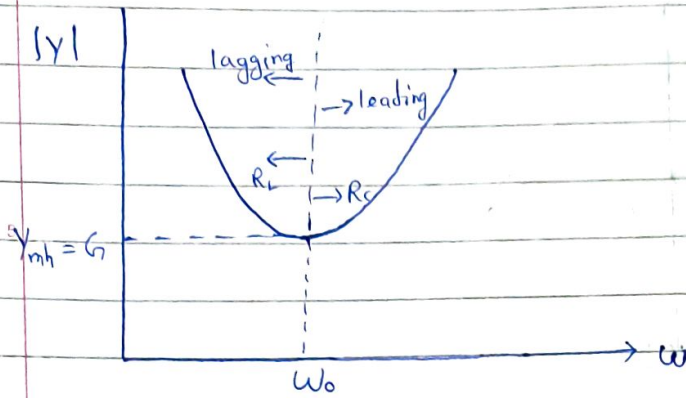
$$\left[\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \right]$$

$$\left[f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \right]$$

$$\gamma = G + j(B_C - B_L)$$

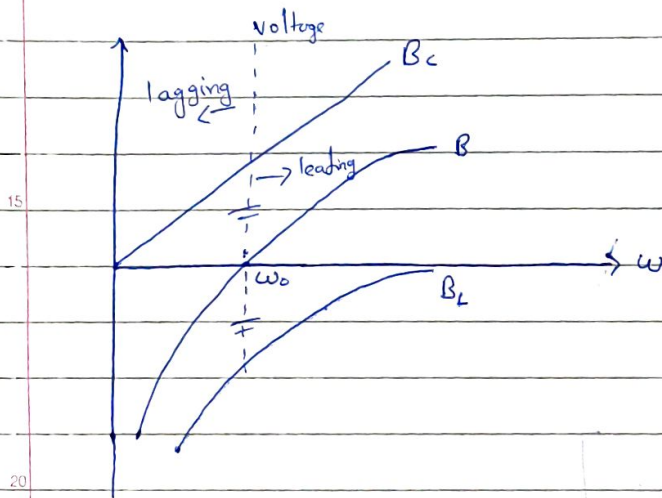
$$= G + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$|Y| = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

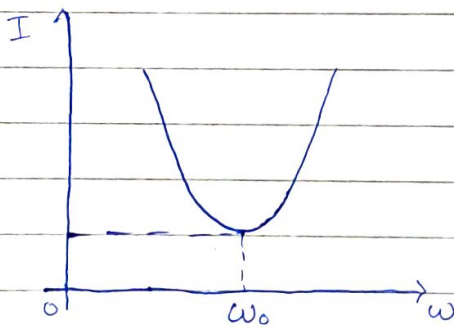


★ $B = B_C - B_L$

$= \omega_C = \frac{1}{\omega_L}$



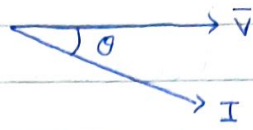
★ I Vs ω curve



★ for $\omega < \omega_0$

$$Y = G - jB$$

↓
RL

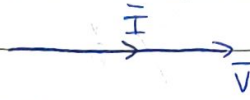


P.f. $\rightarrow$ lagging

* At $\omega = \omega_0$

$$Y = G$$

P.f. $\rightarrow$ unity

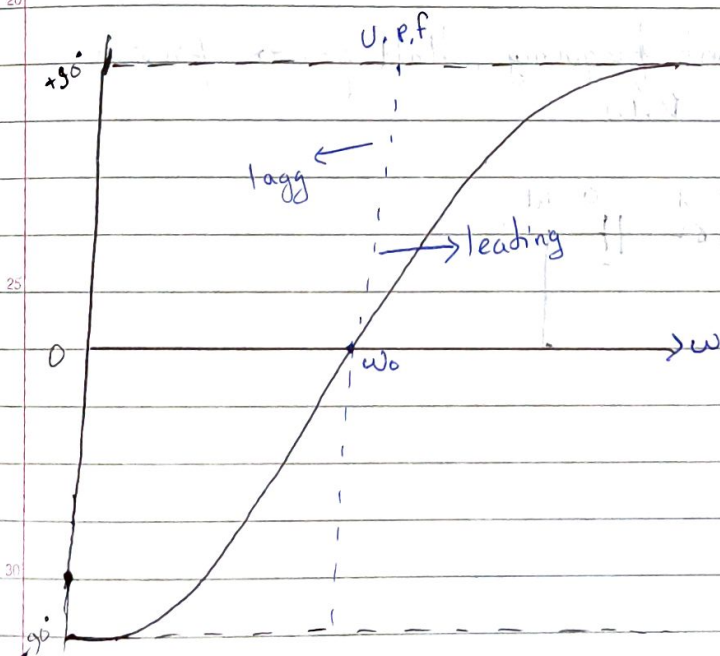
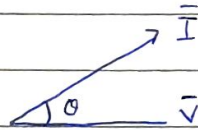


* At $\omega > \omega_0$

$$Y = G + jB$$

↓
RC

P.f. $\rightarrow$ leading



★ Quality factor:-

$$Q = \frac{\text{Reactive component of the current}}{\text{Active component of the current}}$$

$$= \frac{I_L \text{ or } I_C}{I_R}$$

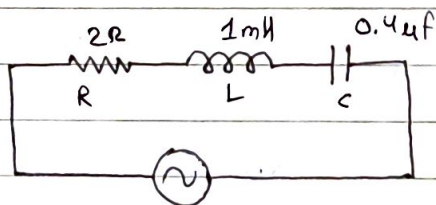
$$Q = \frac{V}{X_L / \frac{V}{I_R}} \text{ or } \frac{V}{X_C / \frac{V}{I_R}}$$

$$\left[Q = \frac{R}{\omega L} \text{ or } \omega R C \right]$$

$$Q = \frac{R}{L} \sqrt{LC}$$

$$Q = R \sqrt{\frac{C}{L}}$$

Qn) ~~Qn~~ Find Resonant frequency, Half power frequency, Quality factor & B.W.



soln Given that

$$R = 2\Omega$$

$$L = 1\text{mH}$$

$$C = 0.4\mu\text{F}$$

$$(1) \omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}}$$

$$\omega_0 = 50 \text{ k rad/sec}$$

$$2) B.W = \frac{R}{L} = \frac{2}{1m}$$

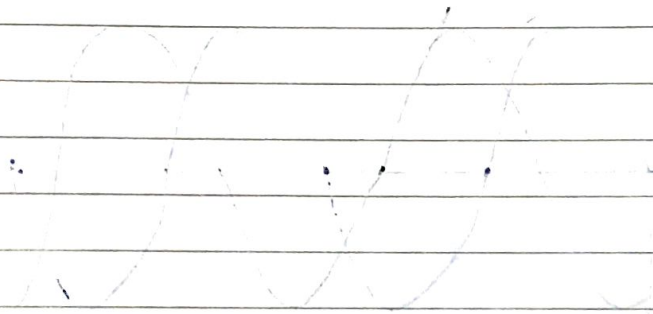
$$B.W = 2 \text{ k rad/sec}$$

$$\star Q = \frac{\omega_0}{B.W}$$

$$\left[\text{or } Q = \frac{\omega L}{R} \quad \omega \Rightarrow \text{Resonant frequency} \right]$$

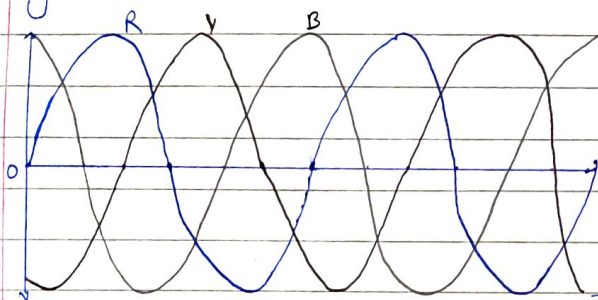
$$Q = \frac{50 \text{ k}}{2 \text{ k}}$$

$$Q = 25$$



Unit-53-phase AC circuitIntroduction:-

- Three phase circuit are important because all the electric power is generated & distributed in 3ϕ
- The three phase circuit has an AC voltage generator also called as alternator that produces 3 sinusoidal voltage that are identical in ~~same~~ shape but phase angle is different
- There is 120° phase difference between all the three phases.
- The electrical energy is transmitted over 3 wire or 4-wire after called lines and N-phase system



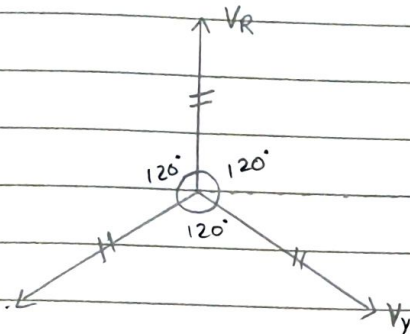
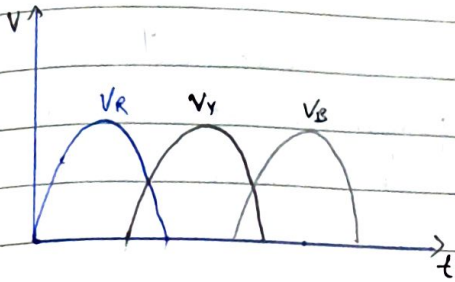
- R
- Y
- B

Phasor Diagram of 3ϕ
sinusoidal waveforms

★ Phase sequences.

It is the order by which the phase voltage attain their peak and the phase sequence may be +ve, -ve

or zero

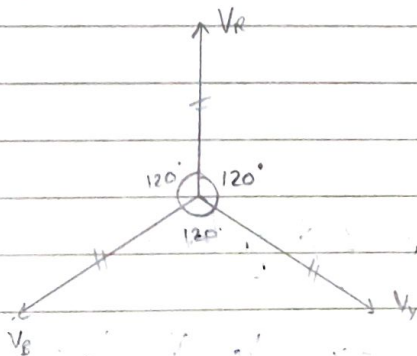
fig:- Phase sequence $\rightarrow$ RYB

$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin(\omega t - 120^\circ)$$

$$V_B = V_m \sin(\omega t + 120^\circ) \text{ or } V_m \sin(\omega t - 240^\circ)$$

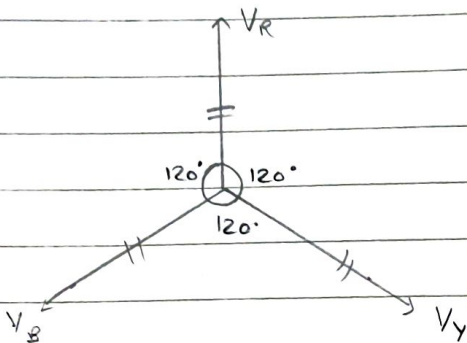
★ Positive phase sequence:-

Phase sequence $\rightarrow$ RYB, YBR, BRV

+ve phase sequence (anti clock wise)

★ Negative phase sequence:-

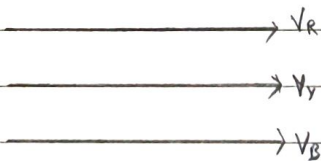
The negative phase sequence is also balanced with three equal magnitude quantities at 120° apart & separately but only when the phase rotation or sequence is reverse.



-ve phase sequence $\rightarrow$ RBY, YRB, BYR
(clockwise direction)

Zero phase sequence:-

That is no particular order of phase sequence means zero sequence



zero phase sequence

★ Balance & unbalanced 3- ϕ system:

- 1) For balanced 3- ϕ system $V_R + V_Y + V_B = 0$, $I_R + I_Y + I_B = 0$
- 2) For unbalanced 3- ϕ system $V_R + V_Y + V_B \neq 0$ & $I_R + I_Y + I_B \neq 0$

★ Advantages of 3- ϕ system

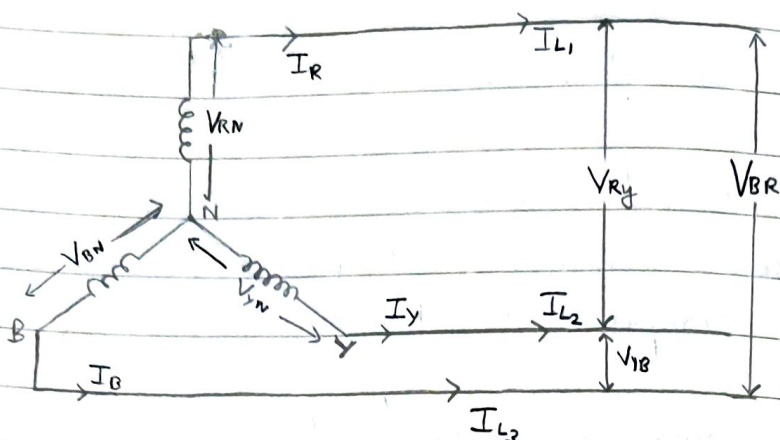
- The amount of conductor material needed to transfer same amount of power is lesser for 3- ϕ system compare to 1- ϕ system.
- Domestic power & industrial power can be provided for same source.
- Voltage regulation of 3- ϕ is better.
- The torque produced by 3- ϕ motor is more.
- With the help of 3- ϕ system interconnection of various network is possible either in star or in delta connection.
- 3- ϕ machine produce less vibration as compared to single phase machine.

★ Types of 3 ϕ connection:-

- 1) Star connection (Y)
- 2) Deltal connection (Δ)

1) Star Connection:-

The below figure shows the balanced 3phase star connected system having sequence



$$|V_{RN}| = |V_{YN}| = |V_{BN}| = V_{ph} \text{ (phase voltage)}$$

(Voltage between a line and neutral)

$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L \text{ or (line voltage)}$$

(i.e. voltage between 2- ϕ)

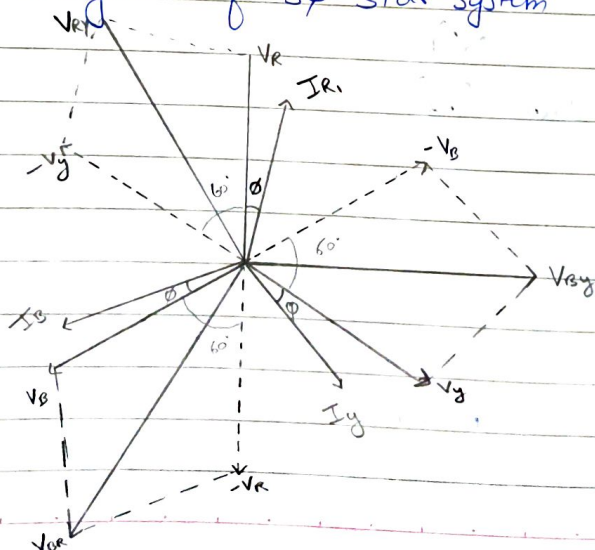
line voltage

$$V_{RY} = V_{RN} - V_{YN}$$

$$V_{BR} = V_{BN} - V_{RN}$$

$$V_{YB} = V_{YN} - V_{BN}$$

Phasor diagram of 3 ϕ star system



Relation between line & phase current

$$I_R = I_Y = I_B = I_{ph} \text{ (phase current)}$$

$$I_L = I_{L2} = I_{L3} = \text{(Line current)}$$

The phase are connected in series so current remain constant.

$$(\text{Line current } [I_L] = \text{phase current } [I_{ph}])$$

Relation between line & phase voltage.

$$V_{RY} = V_R - V_Y \text{ or } V_{RN} - V_{YN}$$

$$V_{YB} = V_Y - V_B \text{ or } V_{YN} - V_{BN}$$

$$V_{BR} = V_B - V_R \text{ or } V_{BN} - V_{RN}$$

V_L (Line voltages)

$$V_{RY} = V_{YB} = V_{BR}$$

line voltage calculation

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2 V_R V_Y \cos 60^\circ}$$

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2 V_{ph} V_{ph} \cos 60^\circ}$$

$$\{ V_L = V_{RY} \}$$

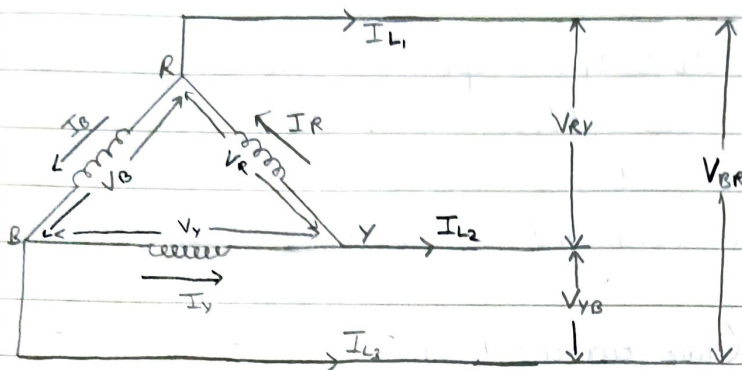
$$V_L = \sqrt{2 V_{ph}^2 + V_{ph}^2}$$

$$\{ V_R = V_Y = V_{ph} \}$$

$$= \sqrt{3 V_{ph}^2}$$

$$= \sqrt{3} V_{ph}$$

★ Delta connection



$$I_R = I_B + I_{L1}$$

$$[I_{L1} = I_R - I_B]$$

$$I_Y = I_R + I_{L2}$$

$$I_B = I_Y + I_{L3}$$

$$[I_{L3} = I_B - I_Y]$$

$$[I_{L2} = I_Y - I_R]$$

★ Relation between line and phase voltage.

$$V_R = V_Y = V_B = V_{\text{phase}}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_L = V_{ph}$$

{ for parallel connection voltage }
remains same

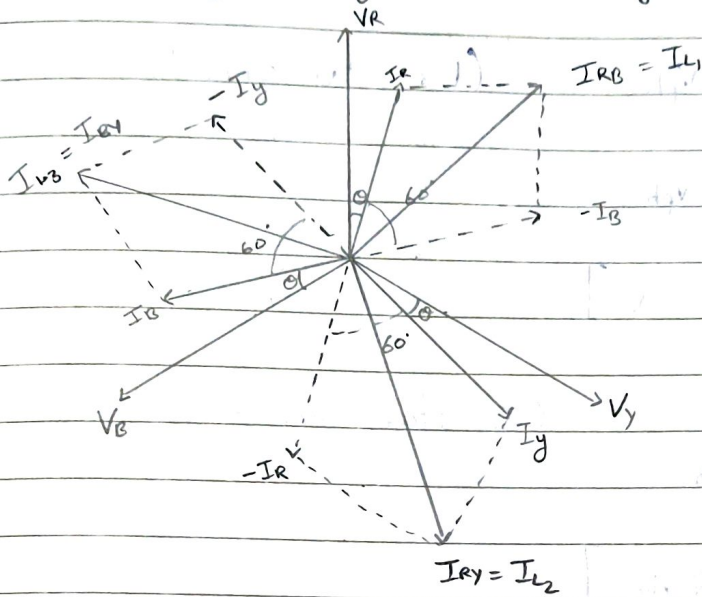
★ Relation between line & phase current.

$$I_{L1} = I_R + (-I_B)$$

$$I_{L2} = I_Y + (-I_R)$$

$$I_{L3} = I_B + (-I_Y)$$

Phasor diagram of 3- ϕ delta system



From phasor diagram

$$I_L = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^\circ}$$

$$= \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \times \frac{1}{2}}$$

$$= \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2}$$

$$= \sqrt{\cancel{3} \times I_{ph}^2}$$

$$[I_L = \sqrt{3} I_{ph}]$$

Q) In a three phase star connected generator the line voltage is 440V. The line current is 15.7 Ampere find the phase voltage & phase current.

⇒ 1) Star connection

2) Line voltage (V_L) = 440V

3) Line current (I_L) = 15.7 Amp

$$V_L = \sqrt{3} V_{ph} \quad (\text{for star connection})$$

$$440 = \sqrt{3} V_{ph}$$

$$[V_{ph} = 254.03 \text{ V}]$$

$$I_L = I_{ph} \quad (\text{In star connection})$$

$$15.7 = I_{ph}$$

$$[I_{ph} = 15.7 \text{ Amp}]$$

② In a 500V, 3 phase Induction motor the phase voltage is 500V & line current is 22.5 Amp line current the Induction motor starter is connected in delta find the line voltage & phase current

⇒ Given that,

Connection type: Δ (delta)

2) Phase voltage = 500V, line current = 22.5 Amp
(V_{ph}) (I_L)

$$I_L = \sqrt{3} I_{ph} \quad (\text{for delta connection})$$

$$\frac{22.5}{\sqrt{3}} = I_{ph}$$

$$[I_{ph} = 12.99 \text{ Amp}]$$

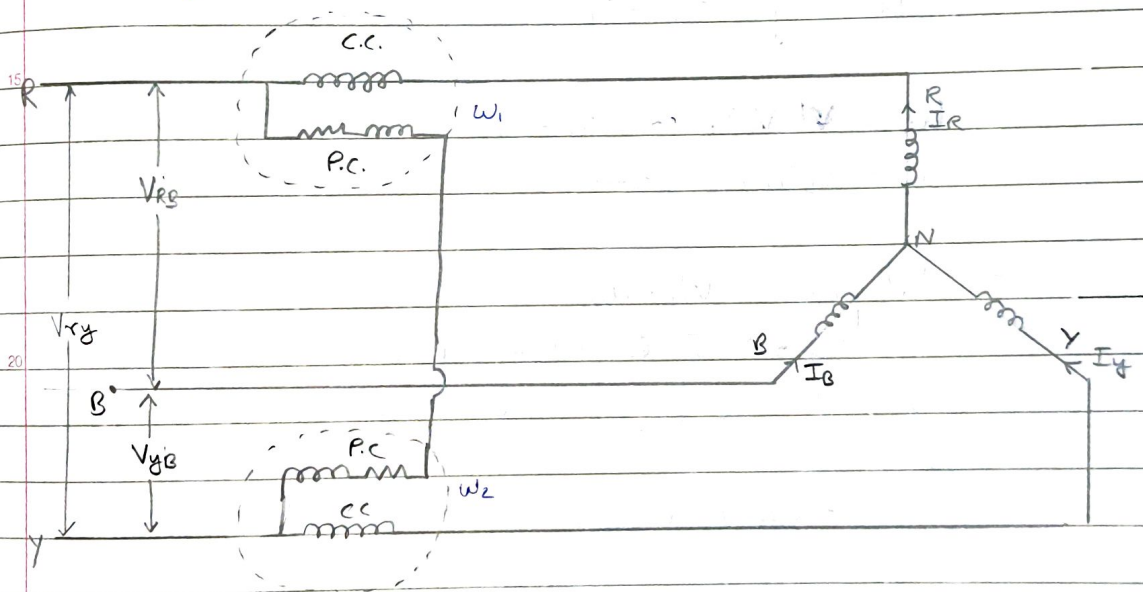
For delta connection

$$V_L = V_{ph}$$

$$[V_L = 550V]$$

* Measurement of 3- ϕ system & power factor of balanced load using 2 wattmeter method.

The 3 ϕ balanced voltage is applied on a balanced three phase load when the current in each phase lag by an angle ϕ behind the corresponding phase voltage & wattmeter readings are:-



P.C. = Potential Coil

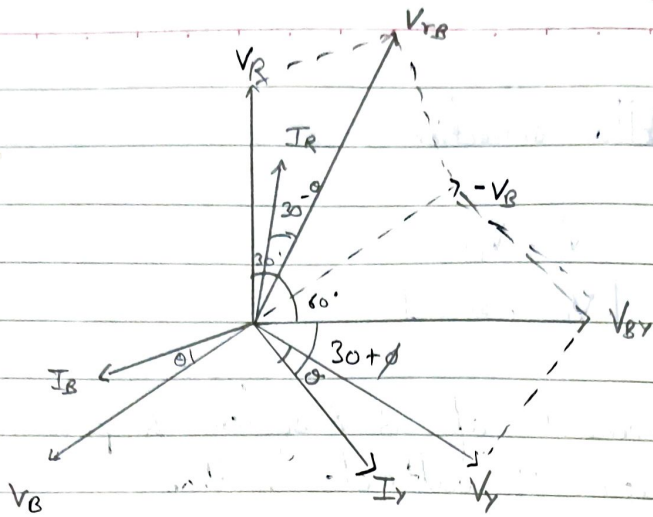
C.C. = Current coil

W_1, W_2 = Wattmeters

I_R, I_Y, I_B = Phase current

$$W_1 = V_{RB} I_R \cos \angle \text{btwn } (V_{RB} \text{ and } I_R)$$

$$W_2 = V_{YB} I_Y \cos \angle \text{btwn } (V_{YB} \text{ and } I_Y)$$



$$W_1 = V_{RB} I_R \cos(30 - \phi)$$

$$W_2 = V_{BY} I_Y \cos(30 + \phi)$$

$$V_{RY} = V_{BY} = V_{BR} = V_L$$

$$I_R = I_Y = I_B = I_L$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$\text{Total power} = W_1 + W_2$$

$$W_1 + W_2 = V_L I_L \{ \cos(30 - \phi) + \cos(30 + \phi) \}$$

$$W_1 + W_2 = V_L I_L \{ \cos(30 - \phi) + \cos(30 + \phi) \}$$

$$\left[\because \cos(A - B) + \cos(A + B) = 2 \cos A \cos B \right]$$

$$\text{So, } W_1 + W_2 = V_L I_L \{ 2 \cos(30) \cos \phi \}$$

$$= V_L I_L \{ 2 \times \frac{\sqrt{3}}{2} \cos \phi \}$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi \quad \text{--- (1)}$$

(Total active power)

Now, Total power = $w_1 - w_2$

$$\Rightarrow V_L I_L \cos(30^\circ - \phi) - \cos(30^\circ + \phi)$$

$$\Rightarrow V_L I_L [\cos(30^\circ - \phi) - \cos(30^\circ + \phi)]$$

$$\Rightarrow V_L I_L \{ 2 \sin 30^\circ \sin \phi \}$$

$$\left\{ \because \cos(A+B) - \cos(A-B) = 2 \sin A \sin B \right\}$$

$$\therefore \Rightarrow V_L I_L \left\{ 2 \times \frac{1}{2} \sin \phi \right\}$$

$$\Rightarrow V_L I_L \sin \phi$$

$$w_1 + w_2 \Rightarrow \sqrt{3} V_L I_L \cos \phi \text{ --- (1)}$$

$$w_1 - w_2 \Rightarrow V_L I_L \sin \phi \text{ --- (2)}$$

Dividing eqn (2) from eqn (1)

$$\frac{w_1 - w_2}{w_1 + w_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\frac{w_1 - w_2}{w_1 + w_2} = \frac{1}{\sqrt{3}} \tan \phi$$

$$\tan \phi = \sqrt{3} \left[\frac{w_1 - w_2}{w_1 + w_2} \right]$$

$$\phi = \tan^{-1} \left[\sqrt{3} \left(\frac{w_1 - w_2}{w_1 + w_2} \right) \right]$$

$$\text{Power factor} = \cos \phi$$